GLINT NO. 330
August 14, 1969

DECLINATION DRIVE-POWER BUDGET STUDY

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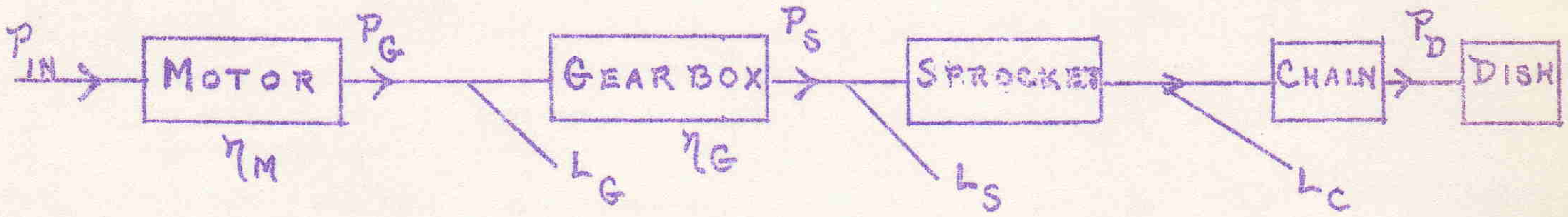
Contents

Motor Power Versus Declination

The following model of the declination drive will be adopted.



P_{IN} = electrical power to motor

η_M = efficiency of motor; adopt 0.8

P_G = power to gearbox $\qquad P_G = \eta_M P_{IN}$

η_G = efficiency of gearbox

L_G = power loss at high speed end of gearbox, independent of load

P_S = power to drive sprocket $\qquad P_S = \eta_G(P_G - L_G)$

L_S = power loss in sprocket friction

L_C = " " " chain

P_D = power to dish $\qquad P_D = P_S - L_S - L_C$

The electrical power to the motor can be measured and the power to the dish can be calculated. Our aim is to assess the various intermediate power losses.

Basic formula

$$P_D = P_S - L_S - L_C$$
$$= \eta_G(P_G - L_G) - L_S - L_C$$
$$= \eta_G(\eta_M P_{IN} - L_G) - L_S - L_C$$

Solving for P_{IN},

$$P_{IN} = \frac{P_D + (L_S + L_C)}{\eta_G \, \eta_M} + \frac{L_G}{\eta_M} \qquad (1)$$

Expected behavior

$P_D = 1.36 \sin\theta$ HP (Appendix 1)

$L_S = 0.50\ (\mu/0.2)$ (Appendix 2)

$L_C = 0.09\ (T/22{,}500)(\mu/0.2)$ (Appendix 3)

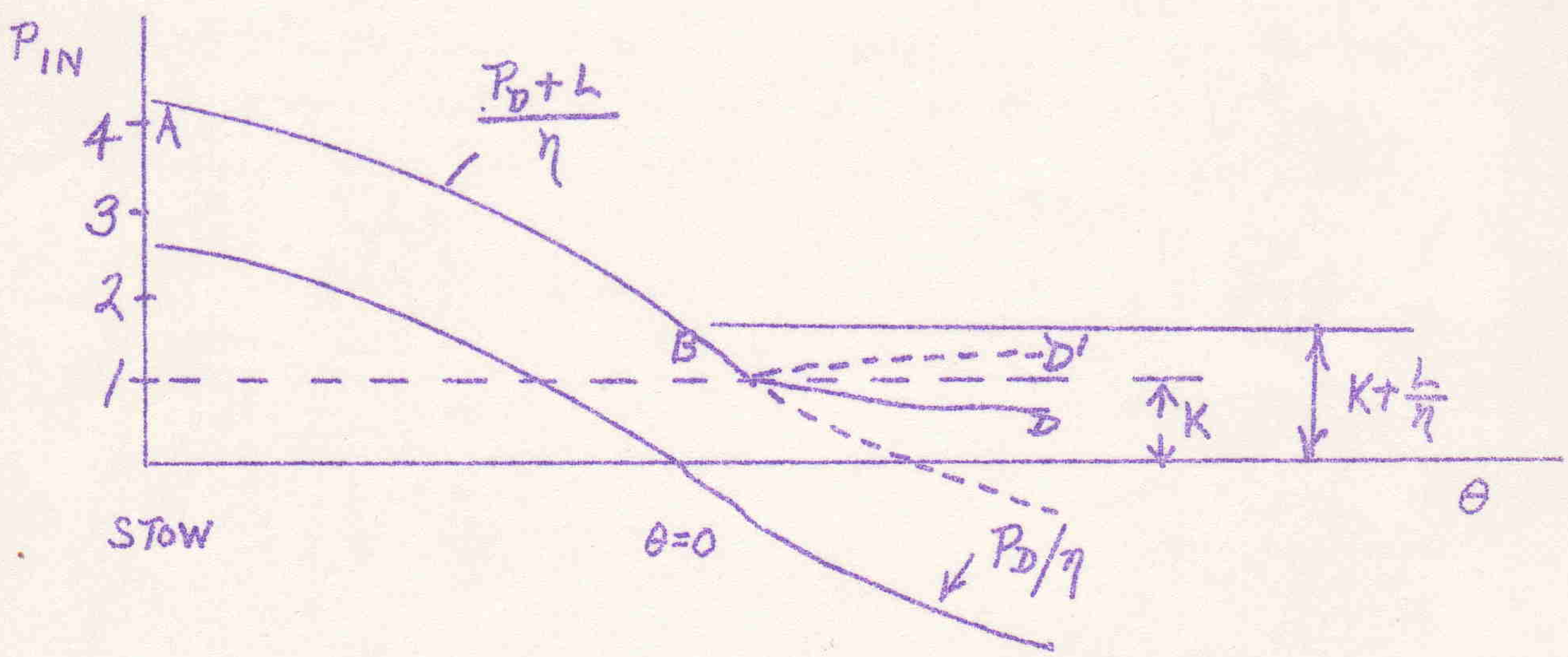
Expected curve of P_{IN} vs. θ

Consider the simple formula

$$P_{IN} = \frac{P_D + L}{\eta} + K \qquad (2.)$$

where $P_D = 1.23 \sin\theta$. This is for raising. When lowering the dish, i.e., when $P_D + L$ is negative, we expect

$$P_{IN} = K + \eta_{REV}(P_D + L) \qquad (3)$$



The expected curve is ABCD. At B, where the dish is balanced, $P_{IN} = K + L/\eta$, so power is still required to move the dish. At C, where $P_D + L = 0$, only the power K is required. On the segment CD, $P_D + L$ is negative so that motion of the dish feeds power back into the gearbox. Then Eq. (3) applies. Now η_{REV} is negative for a self-locking gearbox; the more the dish helps the harder we have to work to lower it (CD′). (See Appendix 5.)

Actual observations

Measurements made on 30 June 1969 (Fig. 1, book 1248) obey Eqs. 2 and 3 in that the observations during lifting can be fitted by a sinusoid and that a break occurs beyond the balance point $\theta = 0$.

Warming up was noticed over half an hour or so, affecting the value of K which dropped from about 1.3 (10 min. after start) to 0.9 HP after 3 trips to the south horizon. Warming up could be accounted for without change in η. The break point did not change.

The displacement of the breakpoints is 1.2 revs of the sprocket or 20° of θ. From Eq. (2), $P_D + L = 0$ at $\theta = 20^\circ$, or $L = 1.23 \sin 20^\circ = 0.42$ HP.

This is a heavy loss; thus at lift-off only 3/4 of the power entering the sprocket shaft reaches the dish. The conclusion is compatible, however, with Appendixes 2 and 3, where, assuming $\mu = 0.2$ we calculate $L_S + L_C = 0.50 + 0.09 = 0.59$ HP.

The value $L = 0.42$ HP seems to be definitely established by observation.

We now compare

$$P_{IN} = \frac{P_D + 0.42}{0.35} + 0.9 \qquad (4)$$

with the measurements and a very good fit results (Appendix 4).

Power Budget Summary

The reliable values for P_D, $P_S + P_C = 0.42$, and P_{IN} lead to the following situation at lift-off after warm-up (third run).

Glint No. 330-5
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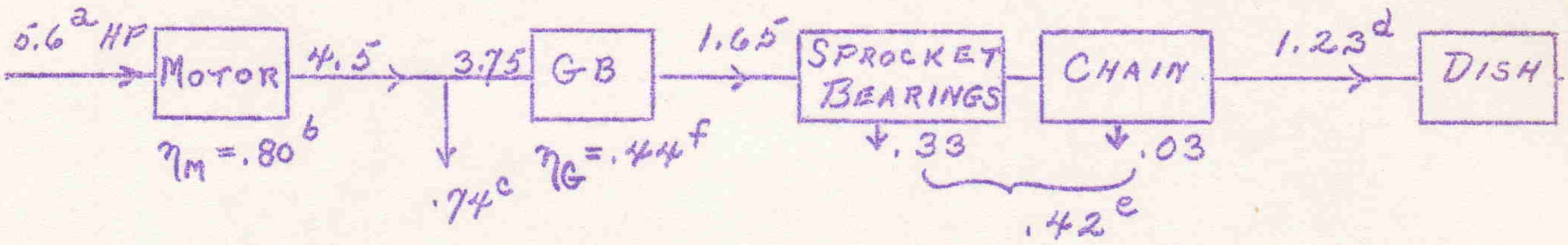


Notes

a. Measured at lift-off, warmed up

b. Estimated

c. Measured lowering dish, warmed up. When cold, value is 1.06, which would require motor power of 1.3 HP. Measured no-load motor power was 1.5 HP (AGL/RSC June 11, '69)

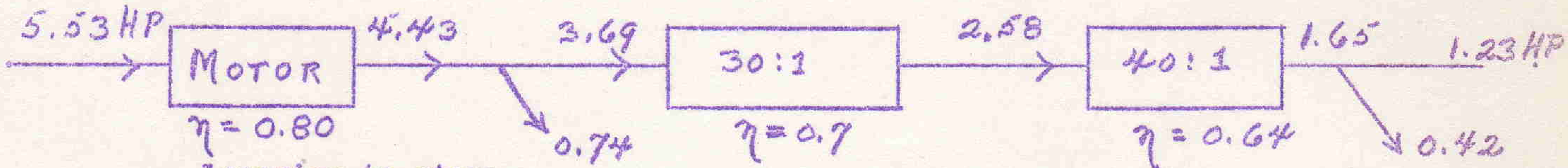
d. Calculated from mass and geometry of dish

e. Deduced from break point in power-declination curve. Indicates μ = .13 in sprocket bearings. Consistent with dish-dropping. Can be cross-checked by measuring static friction

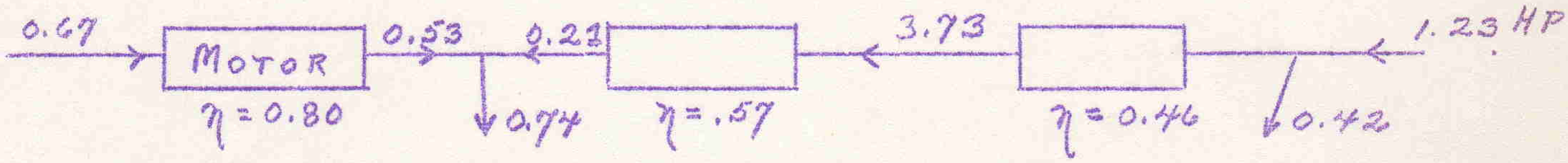
f. Required for consistency.

Detailed summaries based on Appendix 5

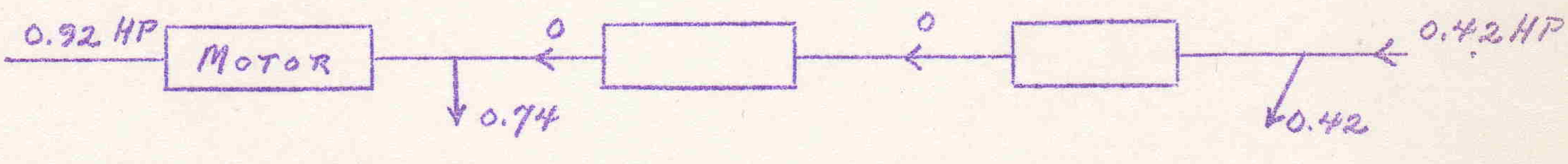
Lifting from stow:



Lowering to stow:

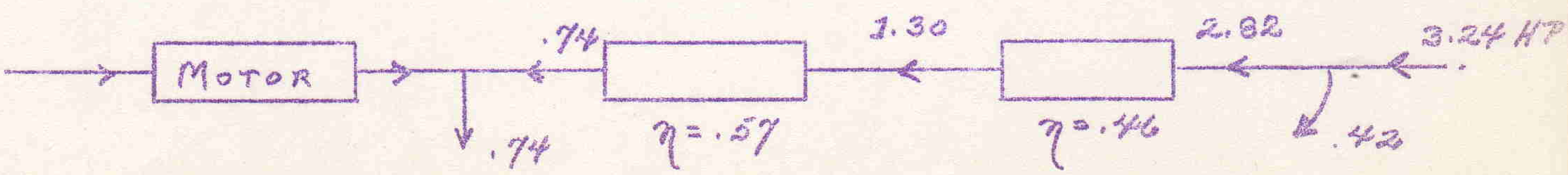


Lowering from break point:



Relief to motor, from breakpoint to stow = 0.92 - 0.67 = 0.25 HP = 0.19 KW

Runaway point:



Discussion

The gearbox behaves as though it dissipates 0.74 HP regardless of load, but subject to warmup, and transmits 0.44 of the remaining input power. This is an overall efficiency at lift-off of 1.65/4.5 = 0.37.

Link Belt Book 2824, p. 45 gives for size 1000, ratio 1180:

Thermal in HP	2.35	
Mech in HP	2.35	Overall efficiency 0.54
Mech out HP	1.26	
Output torque	81,200	
Output r.p.m.	1	

If the incremental efficiency η_G were 0.55, then $P_S = 1.65$ becomes 0.41 and $P_S + P_C = 0.42$ becomes 0.83, doubling the sprocket bearing friction required to a value not compatible with the dish dropping experiment (Appendix 6).

Action suggested by Mr. Bill Henry, Link Belt, San Francisco: Run gearbox at no load for 3 weeks. Warm up transient should be measured weekly, and current monitored throughout.

Appendix 1 - Power to Dish, P_D

$$P_D = -\frac{d}{dt}(W\,R_{CG}\cos\theta)$$

$$= W\,R_{CG}\sin\theta\; d\theta/dt$$

where W = weight of dish = 18,270* lb (Glint No. 293)

R_{CG} = distance of CG from declination axis = 7.6 ft. (Glint No. 293) $= (4.7^2 + 6^2)^{1/2}$

θ = angular departure of CG from balance

$d\theta/dt$ = angular velocity of dish

Motor speed = 1200 r.p.m. very approx.

Gearbox ratio = 1180:1

Sprocket speed = 0.98 r.p.m.

$d\theta/dt$ = 0.98 × (sprocket radius/chain radius)

= 0.98 × (3.845 / 81)

= 0.046 r.p.m. (period = 21 min)

$$P_D = \frac{18{,}270 \times 7.6 \times 2\pi \times 0.046\sin\theta}{33{,}000}$$

$$= 1.23\sin\theta \;\; \text{HP}$$

The origin of θ, as based on calculated CG, is such that the dish is balanced at a declination of 0.7° N.

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* W_1 = 18,270; W_2 = 19,270; $W_3 = W_4 = W_5$ = 20,200. Glint No. 293, p. 2.

Appendix 2 - Power to Turn Sprocket Shafts in Bearings, L_s

The chain tension set by the elastomers is 22,500 lb. (2.8 inches compression at each end of chain). The load F on the drive sprocket bearings is twice this (45,000 lb). The chain tensions under the load W at lift off are 22,500 ± W/2, and the loads on the idler sprocket bearings are $\sqrt{2}$ (22,500 ± W/2), where

$W = 81\ W_1/91$

W_1 = 18,270 lb (weight of No. 1 dish)

91″ = distance of CG from dec. axis (ins.)

81″ = radius of chain (ins.)

Calculate the power from

$P = \pi\mu\ FDN/33{,}000$

where N = 1 rpm

μ = 0.2 (pessimistic dry friction value)

D = shaft diameter in feet.

	F	D	P
Driver bearings	45,000 lb	3 3/4″	0.26
South idler bearings	44,000	2 7/16″	0.17
North idler bearings	19,000	2 7/16″	0.07
			0.50 HP

Appendix 3 - Power to Flex Declination Chain, L_C

Observational data

A pendulum was made in which the hinge was one pin of RC 120 chain and the decay time to σ to half amplitude was measured as follows.

No. of strands	6	1
Weight of bob	200 lb	1000 lb
Pendulum length	65″	81″
Period T	2.5 sec	2.9 sec
$g(T/2\pi)^2$	60″	81″

For initial displacements d between 3 and 10 inches both sets of data were fitted within 20 per cent or so by

$$\sigma = 4d \qquad \text{(d in inches, } \sigma \text{ in seconds)}$$

[see Lab. Notebook 1248, RSC]

Decay time and energy loss

In exponential decay, the decay time to half amplitude is 0.7 time constants. The fractional energy loss in one period, ϵ, is twice the fractional loss of amplitude in one period, i.e.

$$\epsilon = 1.4\, T/\sigma$$

(If $x = x_o \exp(-t/\tau)$, $x_o^{-1}(dx/dt)T = -T/\tau$)

Putting $\sigma = 4d$,

$$\epsilon = 0.35\, T/d$$

When $d = 16''$, the initial displacement of 11 1/4° agrees with the 22 1/2° motion in a link engaging with a tooth. So

$$\epsilon = 0.35 \times 2.9/16 = 0.06$$

With this 6% energy loss per period (two flexings) remain the same when the time taken per flexing is 1/16 min (sprocket tooth spacing). instead of the 1.45 sec (= 2.9/2) as in the pendulum, or will the different speed of flexing (2.6 times slower make a difference? An answer to this is given by the fact that the decay time σ is found to be proportional to initial displacement d. The initial stored energy is proportional to d^2. If the friction loss of energy is proportional to d (or to the relative displacement of the sliding surfaces) regardless of speed of sliding), then σ is proportional to d, as observed. But if viscous friction applies, with friction force proportional to velocity and therefore proportional to d, then the friction loss of energy (force x distance) is proportional to d^2 and the decay constant would be independent of d.

Will the higher pressure in practise invalidate the result? Only moderate change was noted in going from 200 lb. on 6 strands to 1000 lb. on one strand, an increase of 30 times in pressure. Operating pressures produced by 20,000 lb. on 6 strands are only 3 times higher than reached in the test.

Energy loss per flexing

Take the chain tension to be 22,500 lb. (the pretension). (At lift-off, the tensions are 32,000 and 13,000 lb. on opposite sides of the drive sprocket.) The stored energy in a pendulum with a 22,500 lb. bob and length 81" displaced 11½° is $Wd^2/2l$ = 22,500 x ½ x $(11/57)^2$ x (81/12) = 3000 ft. lb. The loss per cycle is 6% or 3% per flexing, say 90 ft. lb. per flexing.

Power loss in chain

= (loss per flexure x flexings/minute)/33,000
= 90 x 6 x 16/33,000
= 0.26 HP.

This loss is proportional to the pretension and will be higher when super-tension is applied for the purpose of keeping the chain taut in wind.

Calculated from dry friction (= 0.2)

Tangential friction force on pin = μ x 22,500 lb. = 4500 lb.

Relative motion ($22\frac{1}{2}°$) = pin circumference/16 = π x 0.437/lb. = 0.085"

Work done per flexing = 4500 x 0.085/12 = 32 ft. lb.

No. of flexings per minute = 6 x 16 = 96 (at 1/16 r.p.m.).

Power loss = 32 x 96/33,000 = 0.09 HP

This may be in agreement with the 0.26 HP above within the accuracy of the latter. But the chain then seems to be acting as if unlubricated. Furthermore, even the low pressure pendulum test (200 lb. on 6 strands) gave losses consistent with a single coefficient of friction.

Conclusion

The chain power loss is negligible. But, why is the chain friction in the pendulum test so high? A high effective value of friction might result from rubbing on a radius larger than the pin radius such as could have been caused by not hanging the bob centrally on the pin.

Appendix 4 - Computed Motor Power, P_{IN}

$$P_{IN} = \frac{P_D + 0.42}{0.35} + 0.93 \text{ HP} \qquad (4)$$

$$= 0.746\ (1.23 \sin\theta + 0.42)/0.35 + 0.671 \text{ kw}$$

$$= 2.63 \sin\theta + 0.89 + 0.7$$

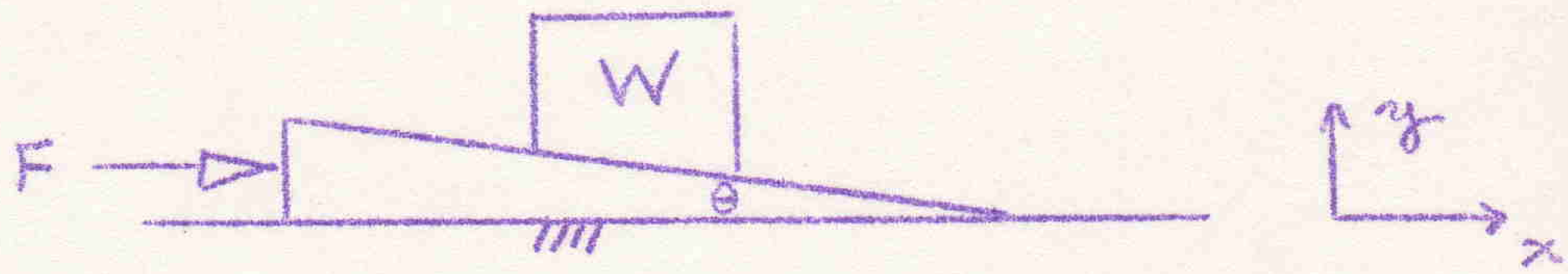
$$= 2.63 \sin\theta + 1.59 \text{ kw}$$

θ	P_{IN}(calc.)	P_{IN}(meas.)
-17°	.83	.85
0	1.6	1.6
17	2.37	2.4
34	3.08	3.1
51	3.65	3.7
68	4.05	4.1
85	4.25	4.25

Appendix 5 - Efficiency of Worm Drive

Lossy wedge

An ideal worm gear contains a dissipative wedge which is specified by its angle θ and coefficient of friction μ. The actual gear box has other complications. Following is the theory of the lossy wedge.

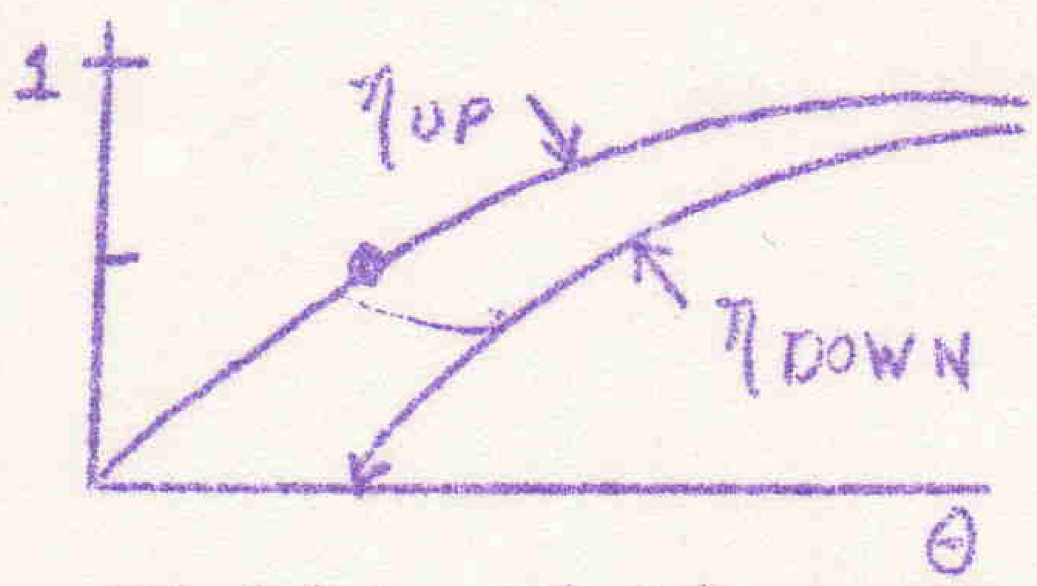


Lifting:

$$F\,dx = W\,dy + \mu W\,dx$$

$$F = W(\tan\theta + \mu)$$

$$\eta_{UP} = W\,dy/F\,dx = 1/(1 + \mu\cot\theta)$$



Lowering:

$$W(-dy) = F(-dx) + \mu W(-dx)$$

$$F = W(\tan\theta - \mu)$$

$$\eta_{DOWN} = \frac{F(-dx)}{W(-dy)} = 1 - \mu\cot\theta$$

$$= 2 - 1/\eta_{UP}$$

If η_{DOWN} is negative, i.e., if the lifting efficiency is 0.5 or less, the wedge is locked against reverse push. To extract the wedge by pulling in the -x direction will be hindered in proportion to W as indicated by the negative value of η_{DOWN}.

Lossy worm

T_i, T_o = input and output torques

r_i = radius of worm

r_o = radius of worm wheel

α_i, α_o = input and output shaft rotations

θ = lead angle of worm

μ = coefficient of friction

f = normal force at contact = T_o/r_o

$R = d_{\ i}/d_{\ o} = (r_o/r_i)\cot\theta$

Input work = Friction loss + Output work

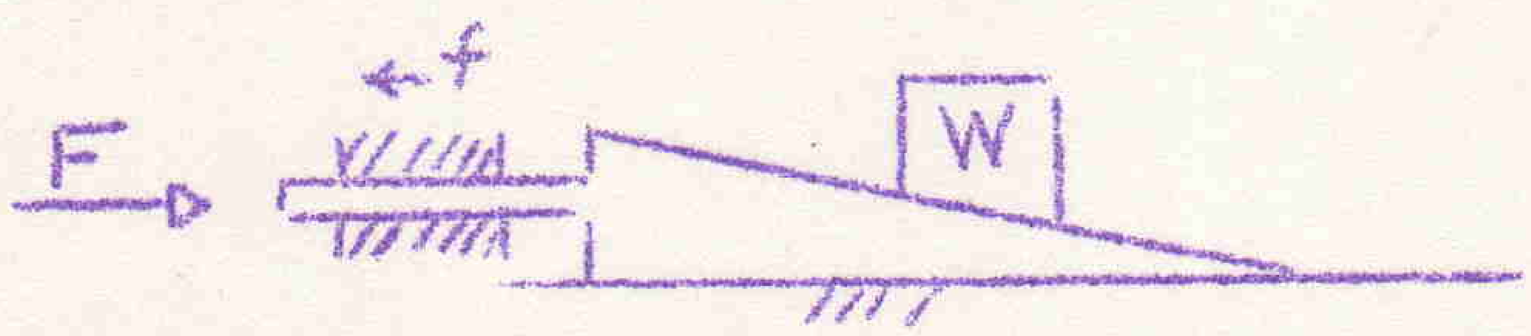
$$T_i\, d\alpha_i = \mu f\, r_i\, d\alpha_i + T_o\, d\alpha_o$$

$$= T_o(\mu r_i\, d\alpha_i/r_o + d\alpha_o)$$

$$\eta = T_o\, d\alpha_o/T_i\, d\alpha_i = 1/(1 + \mu\cot\theta)$$

$$= 1/(1 + R\mu r_i/r_o)$$

Wedge with no-load Loss



A drag force of f is present regardless of load

Lifting

$$(F - f)\, dx = W\, dy + \mu W\, dx$$

$$F/W = \tan\theta + \mu + f/W$$

$$\eta_{UP} = W\, dy/F\, dx = \tan\theta/(\mu + \tan\theta + f/W)$$

In the rotary case, $\eta_{UP} = 1/(1 + R\,\mu r_i/r_o + R\,T_f/T_o)$ where T_f is the input drag torque.

Lowering

$$W(-dy) = (F + f)(-dx) + \mu W(-dx)$$

$$F/W = \tan\theta - \mu - f/W$$

$$\eta_{DOWN} = F(-dx)/W(-dy) = (\tan\theta - \mu - f/W)/\tan\theta$$

$$= 2 - 1/\eta_{UP}$$

If $_{DOWN}$ is negative, the wedge is locked against reverse push. This happens when $\eta_{UP} < 0.5$, as in the $f = 0$ case. However, as the load increases, f/W diminishes and η_{UP} improves. A big enough load will drive the wedge backwards unless the wedge is locked when $f = 0$, i.e., unless $\tan\theta < \mu$.

Double Stage Worm Reducer

Take the case analogous to Link-Belt double worm reducer with output ratio 40:1 and input ratio 30:1. Try $r_i/r_o = 1/70$. Then

$$= \quad _o \quad _i$$

$$= \frac{1}{0/70 + 1} \quad \frac{1}{30/70 + 1}$$

$$= 0.64 \quad 0.7$$

$$= 0.45$$

This agrees with the presumed incremental efficiency of 0.44. The reverse efficiencies are

	2 - 1/0.64 = 0.46	output stage
and	2 - 1/0.7 = 0.57	input stage

and the net reverse efficiency is 0.46 0.57 = 0.26.

Appendix 6 - Dropping the Dish

The dish was dropped about one inch onto elastomers on the pedestal and distance versus time oscillograms were obtained by L. D'Addario.

$$W = 18{,}270 \text{ lb.}$$

$$M = 570 \text{ slugs}$$

$$k^2 = 11.3^2 + 7.6^2 = 186 \text{ ft}^2$$

$$I = 570 \times 186 = 106{,}000 \text{ slug ft}^2$$

$$\text{Ang. accel.} = \text{Torque}/I = 18{,}270 \times 7.6/106{,}000 = 1.3 \text{ rad sec}^{-2}$$

$$\text{Linear accel. at CG} = 7.6 \times 1.3 = 9.9 \text{ ft sec}^{-2}$$

If friction is 7000 lb at 81" radius it is 6000 lb at 91" radius (to CG), and acceleration would be reduced to $9.9 \times 12{,}270/18{,}270 = 6.7$ ft sec^{-2}. The value of 7000 lb is based on 0.42 HP loss and requires loss of 0.74 HP at gearbox input followed by 0.44 incremental efficiency.

If the gearbox had 0.55 incremental efficiency, it would give out another 0.4 HP, making the loss in dish system 0.42 + 0.4 HP and the friction force would be 12,000 lb at 91". Then the acceleration would be reduced to 3.3 ft sec^{-2}.

The oscillograms obtained varied from one to another possibly as a result of different initial conditions imposed by friction and the method of dropping. However, the dish always fell through an inch in less than 0.2 seconds (Fig. 2).