GLINT NO. 153
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FIVE ELEMENT X-BAND ARRAY DESIGN FACTORS

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Array Arrangement

The elements are arranged as in Fig. 1. The basic idea of this grouping is to obtain a minimum redundancy in the spacing of the elements. Thus the array extracts the same information as would be extracted (in less time) from a ten element array formed by filling in the vacant spaces. The 75 foot spacing is redundant, occurring twice. See Glint No. 98 for other arrangements of this kind.

Operating Frequency

A protected band exists from 10,680 to 10,700 MHz. The following numerical data are for

$$f = 10{,}690 \text{ MHz}$$
$$\lambda = 2.80441 \text{ cm.}$$

Dimensions

Reflector diameter	= 60 feet	18.2880 m	652.126λ
Element spacing	d = 75 feet	22.8600 m	815.146λ
Extreme spacing	= 675 feet	205.740 m	7336.31 λ

Pattern Details

Beamwidth of dish	7 minutes of arc (about)
Fringe separation	4.21735 minutes of arc

Array beamwith to half-peak, on meridian after removal of redundancy 16.9637 seconds of arc

The latter two quantities are calculated from

$$\frac{180 \times 60/\pi}{d/\lambda} \quad \text{and} \quad \frac{180 \times 60 \times 60}{\pi 9 d/\lambda} \times 0.6033548$$

where $\mathrm{sinc}(0.6033548) = \frac{1}{2}$.

Total Power Mode

If the elements are connected together as in Fig. 2, so that all transmission line lengths are equal, to form a single antenna, the reception pattern is given by

$$\frac{1}{25}\left|\exp(-j9\theta) + \exp(-j7\theta) + \exp(-j5\theta) + \exp(j3\theta) + \exp(j9\theta)\right|^2$$

$$= \frac{1}{25}(2\cos 9\theta + \cos 7\theta + \cos 5\theta + \cos 3\theta)^2$$

$$+ \frac{1}{25}(-\sin 7\theta - \sin 5\theta + \sin 3\theta)^2$$

where

$$\theta = \pi(d/\lambda)\sin\alpha$$
$$d/\lambda = 815.146$$
$$\alpha = \text{ex-meridian angle}$$

This pattern is tabulated and graphed in Glint No. 150. The beamwidth to half-power is 19.1 seconds of arc

Collecting Area

The area of five 60-foot circles is $(5\pi/4)(18.2880)^2 =$ 1313.39 square meters. Therefore, allowing for a directivity factor of about 0.5, the effective area cannot exceed about 657 m^2. Other factors tending to reduce the effective area are aperture blocking by the feed supports and feed, slots and holes in the reflector, imperfection of the paraboloid, and assorted losses. These are not expected to be serious. The mode of operation of the receiving system may make a substantial reduction, however, for example by introducing a reference source half the time.

No Preamplification

If in Fig. 2 a Dicke receiver is connected as shown, there will be transmission attenuation composed of the loss in approximately 380 feet of transmission line (9db) plus the loss in at least 10 bends and junctions (2db) plus the loss in a flexible coupling (3db).

Preamplification

In Fig. 3 five preamplifiers are followed by five mixers and five signals at intermediate frequency are delivered to the central point. Each mixer has a local oscillator which is synchronized with a central oscillator. Details are as follows:

Preamplifier: Tunnel diode Aertech #T7525

Noise Figure: 5db (including second stage contribution.)

Gain: 17db

Bandwidth: > 800 MHz.

DC Voltage: -15v

Mixer, IF Preamplifier: Aertech #Q7122

RF Bandwidth: 8-12 GHz

Gain RF to IF: 19 db

Noise Figure Double Channel: 8.6 db

IF Bandwidth: ±150 MHz

L.O. Power: -5dbm

D.C. Power: -15v @ 9ma

Synchronizing System

A separate Glint will record the details of the synchronizing system whose principle is briefly as follows (Fig. 5):

A crystal oscillator distributes a 320.7 MHz signal by underground cable to each antenna, where it emerges and proceeds by a well insulated run to the focus.

The 10,690 MHz signal from the local oscillator beats with the synchronizing signal in a harmonic mixer, which gives out a frequency F. If the local oscillator is on frequency, F will be 106.9 MHz, but otherwise will be slightly different. This signal is fed to a phase detector (where it is tripled to 3F) together with the 320.7 MHz synchronizing signal. The DC output of the phase detector tunes the local oscillator.

Multiple Beam Principle

The five antennas constitute 10 different two-element interferometers (two of which are of the same spacing). It is assumed here that the redundant spacing will be retained and utilized for monitoring.

Ten separate receivers will record the two-element interferograms on magnetic tape. Then the interferograms will be added in the computer, with suitable time shifts as indicated by phase calibration, to form records that could have been obtained by combining the 5 signals electrically and feeding to a single receiver.

Many independent beams can be formed by different ways of phasing the 5 signals but only one of these can be extracted by one receiver. By using N receivers we can spend N times more time on a source and obtain a signal-to-noise ratio corresponding to as much as $\sqrt{N}$ times more effective area.

Central Combining System

At the central point we require

- Five tapped delay cables
- Five phase-equalized amplifiers
- Five 4-way power dividers
- Ten receivers
- Recording system
- Programmer

As the period of the fastest fringes can be about 2 seconds, means for slowing the fringes can be considered.

The 9 independent receivers give an advantage of a factor of 3.

Figure 4 shows a 10-receiver system. An eleventh receiver would be needed for recording a single dish.

The delay cables cope with the fact that the fringes formed at one extreme of the band do not coincide with those at the other extreme. Taking a half-wavelength difference over the band to be tolerable, then under the limiting condition of a source in line with the antenna array the path difference is 7336 wavelengths at 10,690 MHz and is 7336 $\pm\frac{1}{4}$ at 10,690 $\pm$ 10,690/4 x 7336. The tolerable bandwidth is therefore 10,690/14,672 = 0.73 MHz. The time interval and number of delay increments on the delay devices must be adjusted to allow for the desired bandwidth, and range of hour angles.

Effective Reception Pattern

Each spacing generates a cosine function response. Thus the extreme pair has a response

$$\cos[2\pi(9d/\lambda)\sin\alpha] = \cos 18\theta,$$

which may be derived directly as follows. The voltage received from an ex-meridian direction α arrives early or late, with reference to the midpoint of two elements whose spacing is $9D$, by a time $(4\frac{1}{2}D \sin\alpha)/c$. Thus a plane-wave signal from direction α, which would have been received as $A\cos(\omega t + \phi)$ at the midpoint, is received at the two elements as

$$A \cos[\omega(t \pm 4\tfrac{1}{2}d \sin\alpha/c) + \phi]$$

$$= A \cos[\omega t + \phi \pm 9\theta], \text{ where } \theta = \pi(d/\lambda)\sin\alpha.$$

The equipment multiplies these two voltages generating a product

$$A^2\cos[\omega t + \phi + 9\theta]\cos[\omega t + \phi - 9\theta]$$

$$= \tfrac{1}{2}A^2\cos[2\omega t + 2\phi] + \tfrac{1}{2}A^2\cos 18\theta$$

of which the second term only is preserved.

The computer sums the responses to give an effective reception pattern with 9 terms

$$\cos 18\theta + \cos 16\theta + \cos 14\theta + \ldots + \cos 2\theta$$

$$= \sum_{1}^{9} \cos(2n\theta)$$

$$= \frac{\tfrac{1}{2}\sin 19\theta}{\sin\theta} - \tfrac{1}{2}$$

This formula is similar to the identity

$$2\sum_{n=1,3,5\ldots}^{N-1} \cos n\theta = \frac{\sin N\theta}{\sin\theta}, \text{ N even,}$$

occurring in the theory of multielement arrays, and is proved as follows.

$$\frac{\sin 19\theta}{\sin\theta} = \frac{w^{19} - w^{-19}}{w - w^{-1}} \quad \text{where } w = \exp(i\theta)$$

$$= z^{-9}\,\frac{1 - z^{19}}{1 - z} \quad \text{where } z = w^2$$

$$= z^{-9}[1 + z + z^2 + \ldots + z^{18}]$$

$$= (z^{-9} + z^9) + (z^{-8} + z^8) + \ldots + (z^{-1} + z) + 1$$

$$= 2\cos 18\theta + 2\cos 16\theta + \ldots + 2\cos 2\theta + 1.$$

The constant term in the reception pattern arises from failure to incorporate the response of a single antenna. This requires the eleventh receiver referred to earlier.

The pattern $(\sin 19\theta)/19\sin\theta$ is tabulated and graphed in Glint No. 150. It falls to its first zero where $19\theta = \pi$, or $19\pi(d/\lambda)\sin\alpha = \pi$. This is at $\sin\alpha = \lambda/19d = 0.000645673$ or $\alpha \doteq 13.3$ seconds of arc. This is the peculiar interval for antenna observations at the meridian.

Fringe Speeds

Fringe speeds are in the ratio 1:2:3 ... 8:9. The fastest fringe speed occurs for meridian observations with the outer pair of elements whose response is $\cos 18\theta$. The spatial period is given by $18\theta = 2\pi$ or $18\pi(d/\lambda)\sin\alpha = 2\pi$ whence $\alpha \doteq 206264.8/9(d/\lambda) = 28.1157$ seconds of arc. Allowing (1/15) sidereal seconds per second of arc the fringe periods are

1.87438 seconds
2.108
2.410
2.811
3.374
4.217
5.621
8.435
16.869

Sensitivity

Adopt the following parameters for a sample calculation.

Receiver noise temperature	740°K (5.5db)
S/N	10:1
Effective area	657 m^2
Integration time	600 seconds
Bandwidth	50 MHz
Assorted losses	4 db

Glint No. 153-8
9/19/66

Assume that the average source is 2 minutes of arc wide, so that only one sixth of it is in the beam at any one time. It remains in the beam about one second, and so reappears in a fringe every 16 seconds approximately. Consequently 3 hours tracking is required. (Of course, if it is a point source, and if the multiple-receiver system is in operation, an improvement must be allowed for.) Then the limiting flux density is given by

$$\frac{3 \times 6 \times 10 \times 740k \times 10^{26}}{657\sqrt{600 \times (50 \times 10^{6}}} = 1.65 \text{ flux units}$$

Source List

At a level of 1.65 flux units the sources detectable will be essentially those having flux densities greater than 10 in the Owens Valley Observatory Radio Source Catalog, assuming a spectral index of 0.8. These sources are listed in order of strength.

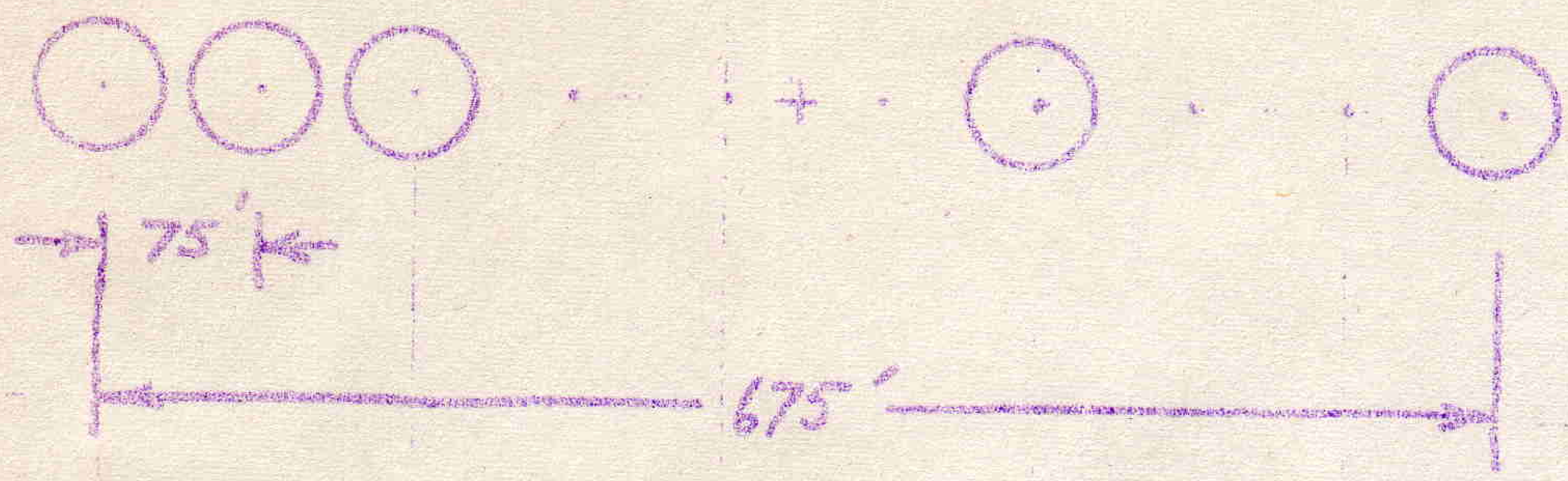

FIG. 1

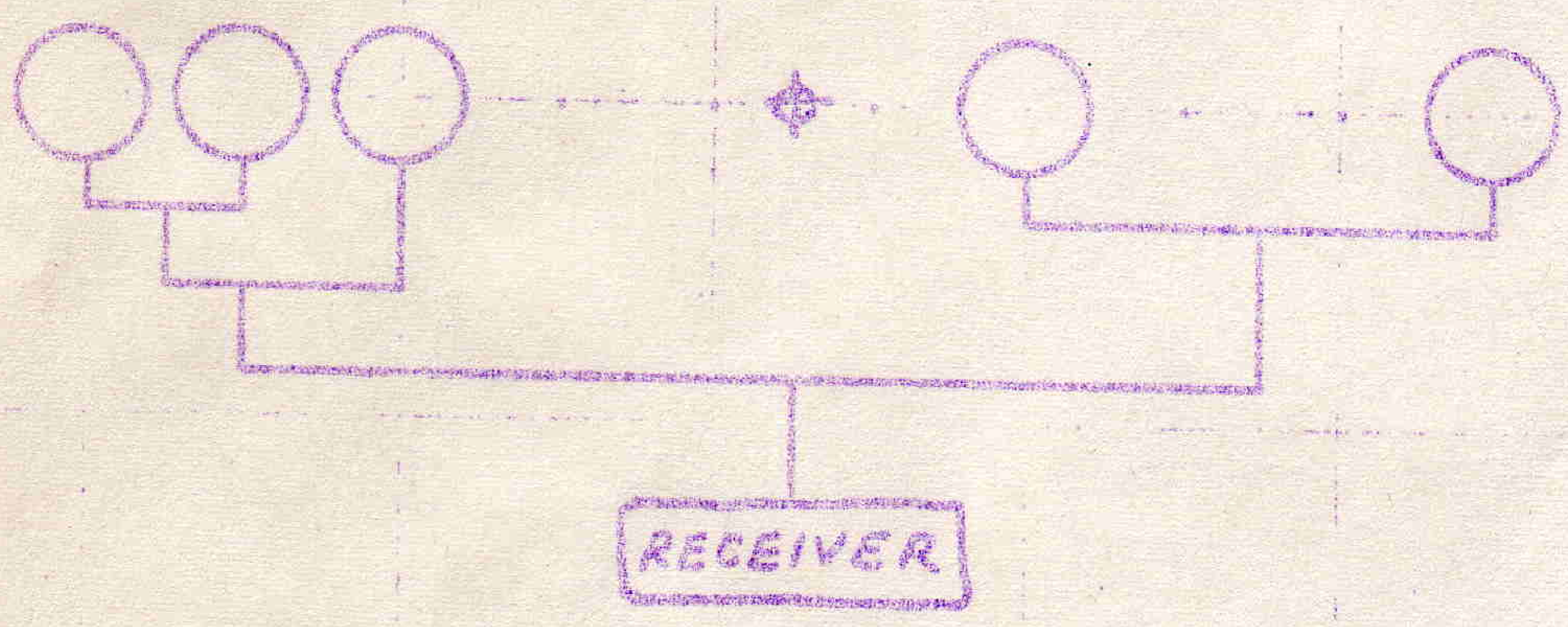

FIG. 2

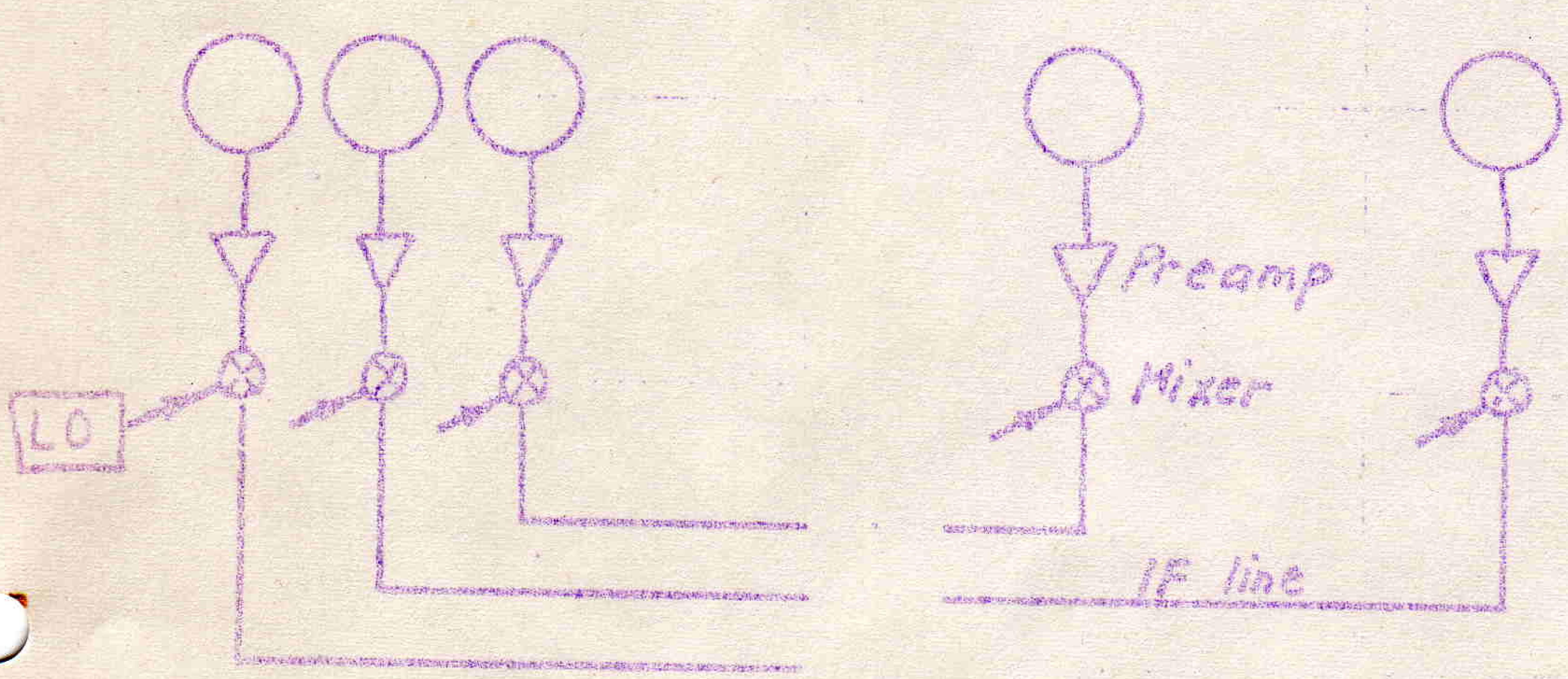

FIG. 3

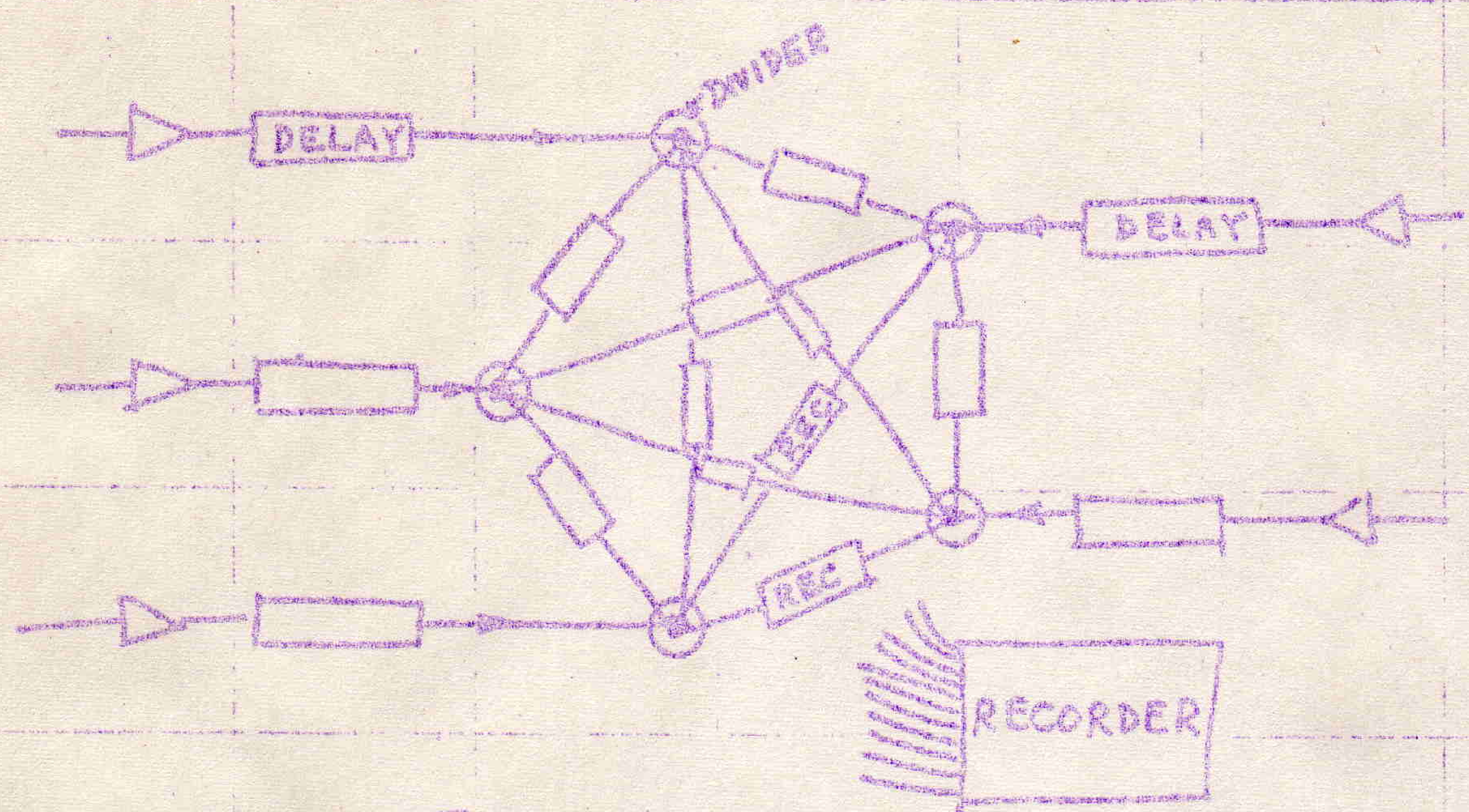
DIVIDER
DELAY
DELAY
REC
REC
RECORDER

FIG. 4

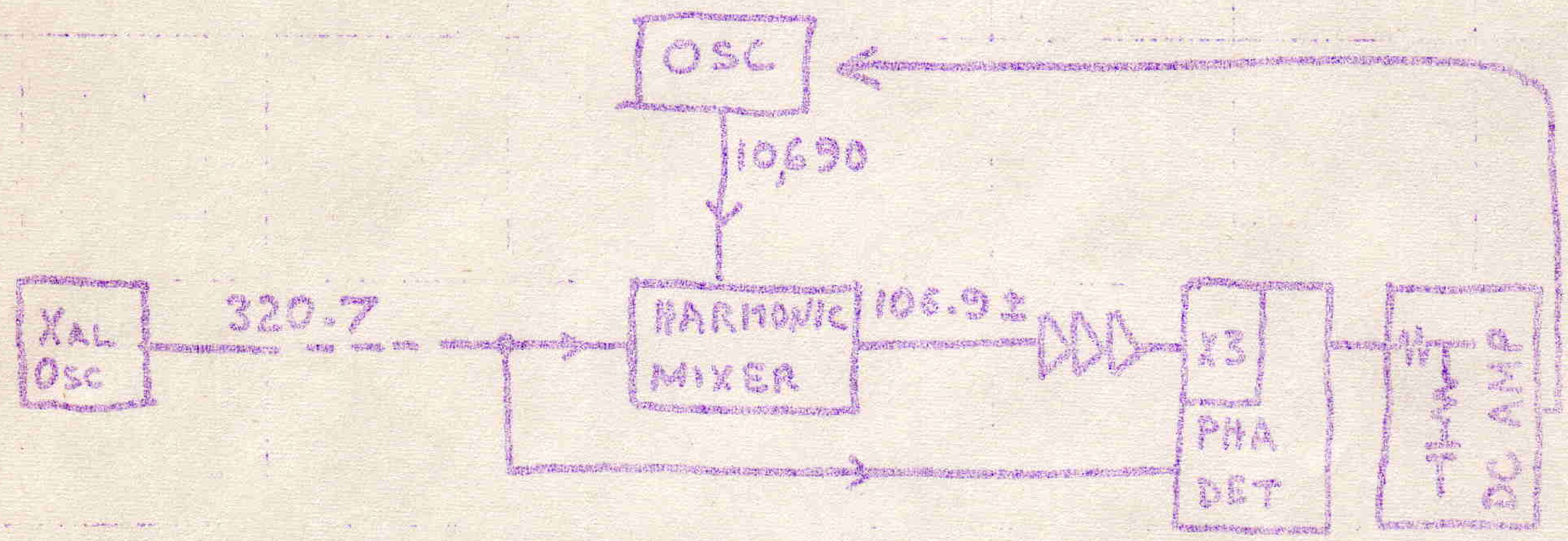
OSC
10,690
XAL OSC
320.7
HARMONIC MIXER
106.92
X3
PHA DET
DC AMP

FIG. 5

OWENS VALLEY OBSERVATORY CATALOGUE EPOCH 1950
SOURCES LISTED IN ORDER OF FLUX MAGNITUDE

SOURCE	R A			DEC. °	′	″	FLUX	
3C461	23	21	9.0	58	32	45	1600.00	Cassiopeia
3C405	19	57	45.0	40	35	46	1700.00	Cygnus
3C144	5	31	31.2	21	59	17	750.00	Taurus
CTB52	16	17	40.0	-16	12	4	450.00	Omega
3C400	19	21	30.0	14	24	-0	436.50	–
3C145	5	32	51.5	-5	25	16	405.60	Orion
3C163	6	29	18.0	5	12	-0	331.70	Rosetta
M13-4/2	13	22	36.0	-42	45	0	300.00	Centaurus
M17-2/13	17	42	36.0	-28	41	0	250.00	Sagittarius
3C274	12	28	18.0	12	39	43	200.00	Virgo
3C157	6	14	36.0	22	43	-0	189.10	IC443
NRAO569	16	32	41.0	-7	22	-0	90.00	
3C390.2	18	44	25.0	-2	33	-0	80.00	
3C400.0	19	20	40.0	14	6	-0	76.00	
3C403.2	19	52	19.0	32	46	-0	75.00	
3C392.0	18	53	38.0	1	15	-0	71.00	
0518-45	5	18	24.0	-45	49	49	66.00	
3C147.1	5	39	11.0	-1	55	29	64.60	
3C353	17	17	55.3	-0	55	44	57.30	
NRAO621	19	59	49.0	33	9	-0	55.00	
3C387.0	18	38	35.0	-5	11	-0	51.00	
NRAO598	19	7	54.9	8	59	9	47.20	
3C123	4	33	55.6	29	34	13	46.60	
NRAO650	21	11	6.0	52	13	-0	46.00	
3C348	16	48	40.3	5	4	23	45.30	
3C 10.0	0	22	37.3	63	51	42	43.50	
3C273	12	26	33.2	2	19	42	43.00	
3C218	9	15	41.3	-11	53	4	42.30	
3C139.1	5	19	21.0	33	25	-0	39.90	
NRAO662	21	27	41.0	50	35	-0	37.00	
NRAO602	19	13	19.0	10	57	-0	35.00	
3C58	2	1	49.0	64	35	14	34.20	
3C398.0	19	8	42.9	8	59	49	32.60	
NRAO572	18	35	32.9	-6	50	18	29.70	
3C153.1	6	6	40.4	20	29	22	29.10	
3C397.0	19	4	56.6	7	1	50	29.00	
CTB32	8	57	41.0	-43	34	0	27.00	
NRAO150	4	0	1.0	51	8	-0	25.70	
3C295	14	9	33.8	52	26	18	22.40	
3C147	5	38	43.5	49	49	43	22.40	

SOURCE	R A			DEC.			FLUX
2356-61	23	56	24.0	-61	11	42	21.00
3C391	18	46	49.0	-0	58	48	20.60
NRAO165	4	7	8.0	50	58	-0	19.00
4C28.56	23	17	25.4	28	11	54	19.00
NRAO579	18	43	30.1	-2	46	39	19.00
3C161	6	24	43.0	-5	51	21	18.90
M18-1/3	18	11	15.4	-17	12	54	18.00
3C270.0	12	16	50.6	6	6	0	17.90
0521-36	5	21	14.0	-36	30	0	16.80
3C358	17	27	40.9	-21	27	11	16.30
3C48	1	34	49.8	32	54	22	15.60
3C286	13	28	49.7	30	45	59	15.30
NRAO584	18	50	52.4	1	9	-0	15.30
3C411	4	15	1.7	37	55	2	14.90
3C380	18	28	13.4	48	42	39	14.70
3C196	8	9	59.4	48	22	5	14.10
3C394	19	1	39.0	5	21	40	14.00
NRAO589	18	59	15.7	1	42	31	13.70
3C84	3	16	28.7	41	19	52	13.50
3C409	20	12	18.2	23	25	46	13.40
1932-46	19	32	19.7	-46	28	12	12.40
3C33	1	6	13.9	13	3	30	12.30
3C390.3	18	45	47.0	79	43	0	12.30
3C433	21	21	30.7	24	51	34	12.20
3C20	0	40	19.5	51	46	53	12.00
NRAO567	18	28	51.6	-2	6	-0	12.00
3C434.1	21	23	26.5	51	42	14	11.60
3C454.3	22	51	29.4	15	53	5	11.40
2104-25	21	4	26.5	-25	39	30	11.40
3C454.1	22	49	36.0	71	36	-0	11.30
NRAO607	19	15	47.0	12	6	-0	11.20
0750-26	7	50	27.0	-26	16	30	11.00
3C452	22	43	33.0	39	25	39	10.70
3C279	12	53	35.8	-5	31	8	10.50
3C410	20	18	4.7	29	33	2	10.50
NRAO601	19	11	59.0	11	3	30	10.20
0902-38	9	2	-0.	-38	25	0	10.00
NRAO596	19	4	41.3	6	30	-0	9.80
3C 66.0	2	19	57.6	42	45	47	9.70
3C98	3	56	11.3	10	17	32	9.70
3C138	5	18	16.5	16	35	26	9.60
4C20.24	10	55	36.0	20	14	45	9.50
3C134	5	1	17.6	38	1	58	9.30
3C396.1	19	4	18.0	-3	6	-0	9.20
3C444	22	11	42.4	-17	16	32	9.20