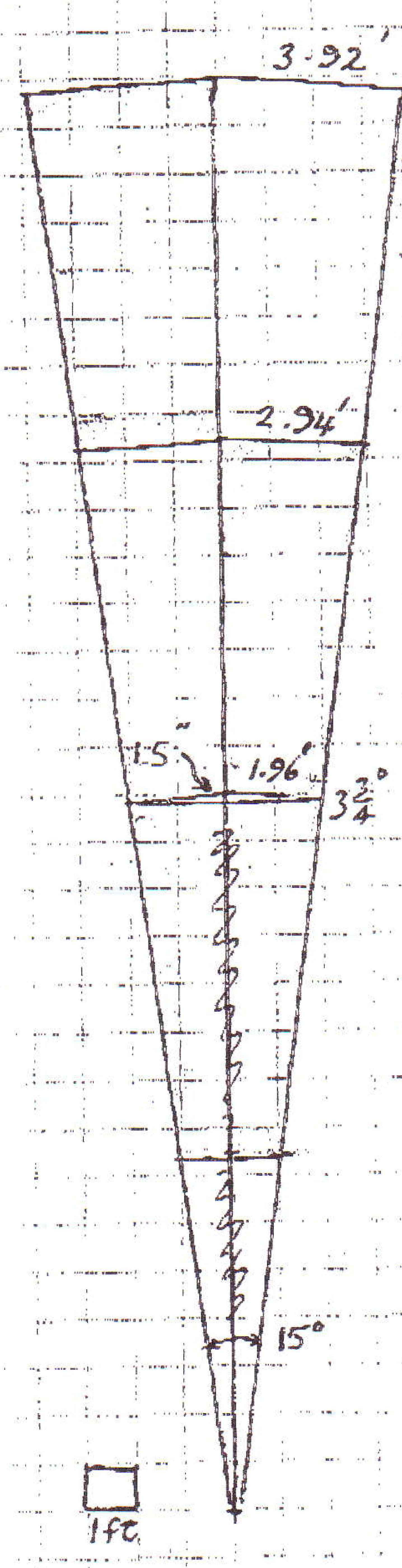
Arrangement of Panels.



		Area ft^2	Approx weight lb
48 Outer panels:	7'6" x 3.92' x 2.94'	1260	1200
48 Middle panels:	7'6" x 2.94' x 1.96'	900	800
24 Inner panels:	7'6" x 3.92' x 1.96'	180	500
			2500 lb

Area estimate does not allow for departure of paraboloid from plane projection or for flanging. Assumes 14^{II} Al. (.063") ~ 0.3 lb/ft^2

Thermal expansion

In 90 inches, .0000124 x 90 = .001 inch/deg F

On a hot day we might get 30°F temp. diff. between sunlit skin and shaded frames giving .03 inches expansion or a strain of $.03/90 = \frac{1}{3000}$. This wd produce a stress of $10^7/3000$ = 3000 lb/in² if restrained, or, on a cross-section of 36" x .062", a load of 6000 lb.

Hence, if the support points of the panel will yield .03 inches under a load of 6000 lb, the panel will not buckle thermally. Putting $\delta = Wl^3/3EI$, $\delta = .03$, $W = 6000$, $E = 3\times10^7$, $I = \frac{1}{12}bd^3$, we have $l^3/bd^3 = 40$. Even a stubby stalk with $b = d = \frac{1}{2}$" and $l = 1\frac{1}{2}$" has this amount of give, but

[redacted]

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Inverse transformation : $x' = ax + bz$

$z' = az - bx$

Ellipses of const y

$$m^2\left[z' - \left(x' + \frac{h}{2ma}\right)\right]^2 + \frac{y^2}{a^2} = (m^2-1)x^2 + \frac{2hm}{a}x' + \frac{h^2}{4a^2}$$

axis ratio $= ma$

* Curvature at $z^* = 0$: differentiate w.r.t. to y

$$m^2[z' - z_0]^2 + \frac{y^2}{a^2} = \text{const}$$

$$\cancel{2}[z'-z_0]\left[\frac{dz'}{dy}\right] + \frac{\cancel{2}y}{m^2a^2} = 0$$

$$\left(\frac{dz'}{dy}\right)^2 + [z'-z_0]\frac{d^2z'}{dy^2} + \frac{1}{m^2a^2} = 0 \qquad \text{at } y=0,\ \frac{dz'}{dy} = 0$$

$$\frac{d^2z'}{dy^2} = \frac{-1}{m^2a^2(z'-z_0)} = \frac{-1}{\text{rad of curv}}$$

Rad. of curvature $= m^2a^2(z' - z_0)$ $\qquad z_0 = \dfrac{h}{2ma} = \dfrac{36}{.552 \times .875} = 74.5\ ft$

$= .234(z' - 74.5)$

≈ 20, should be ~ 40

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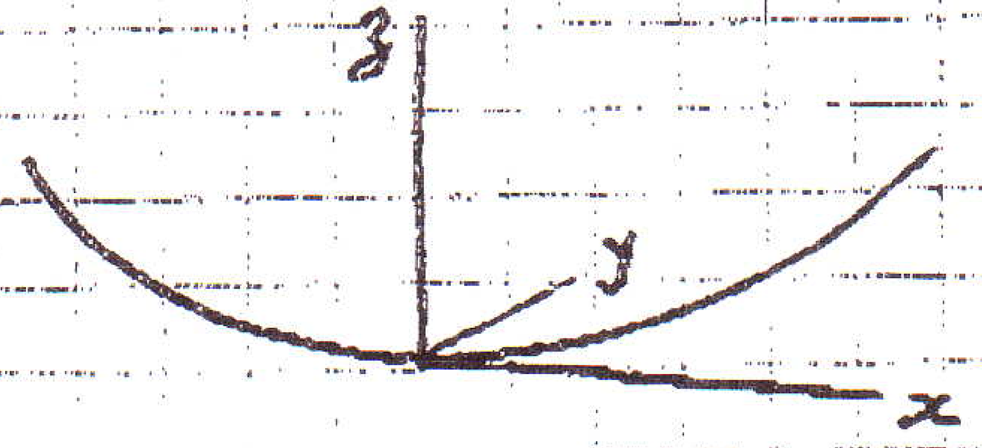
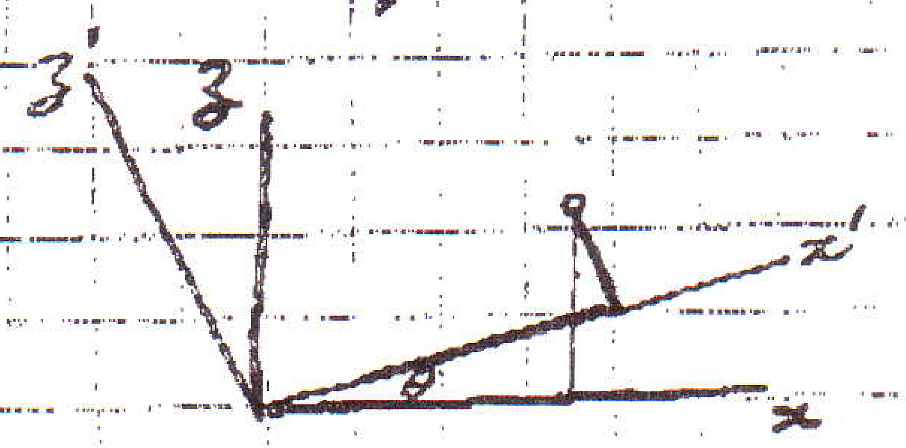
Equation of Paraboloid Referred to Inclined Plane

9 Feb 1965

$$hz = x^2 + y^2 \quad \ldots\ldots (1)$$

$h = 72$ feet





Transformation of coordinates

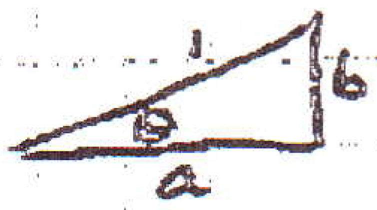
$$\begin{cases} x = a x' - b z' \\ z = a z' + b x' \\ y = y' \end{cases}$$

use K S

$a = \cos\theta$

$b = \sin\theta$

$m = b/a$



Eq. of paraboloid referred to $x'y'z'$, by substitution in (1), is

$$haz' + hbx' = (ax' - bz')^2 + y'^2$$

(dropping the primes) and transposing:

$$a^2x^2 - 2abzx + b^2z^2 + y^2 - hbx = haz \quad (1.1)$$

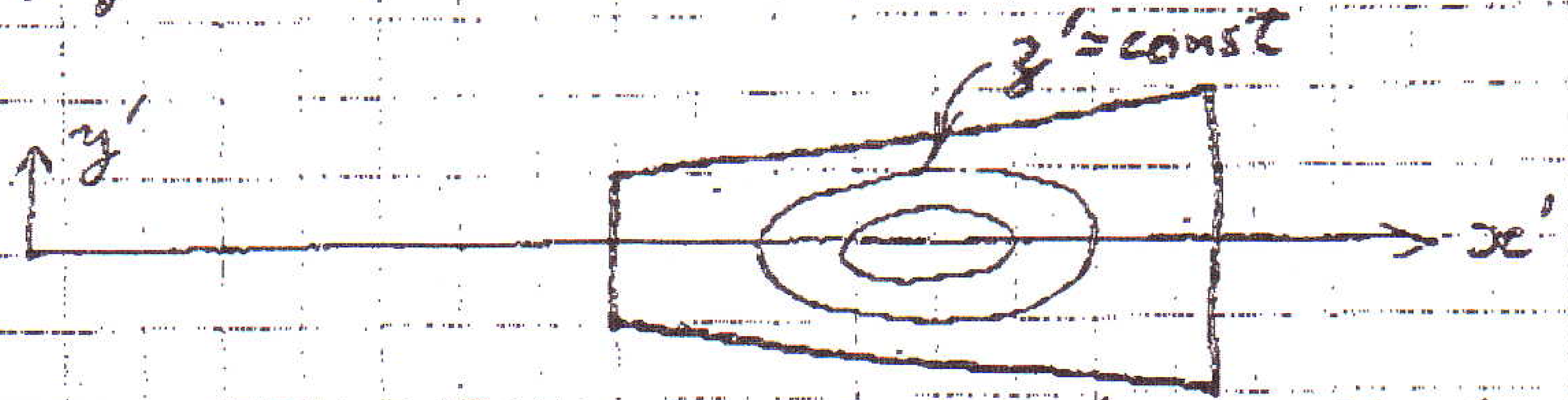
$$x^2 - 2mzx + m^2z^2 + \frac{y^2}{a^2} - \frac{h}{a}mx = \frac{h}{a}z$$

$$x^2 - \left(2mz + \frac{h}{a}m\right)x + m^2z^2 + \frac{y^2}{a^2} = \frac{h}{a}z$$

$$\left[x - \left(mz + \frac{hm}{2a}\right)\right]^2 + \frac{y^2}{a^2} = \frac{h}{a}z + \frac{hm^2}{a}z + \frac{h^2m^2}{4a^2} \quad (2)$$

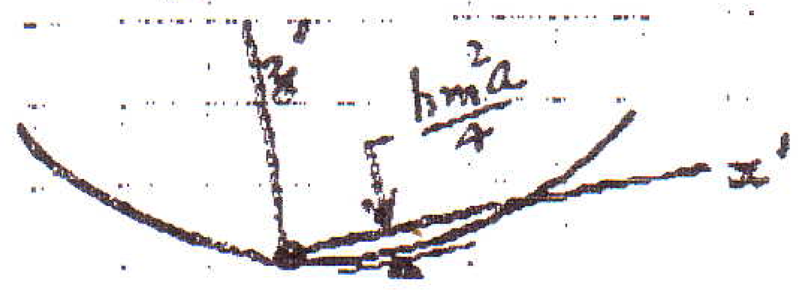
Replace primes

Contours of constant z' are thus ellipses of axis ratio a



Minimum occurs when RHS = 0, $\frac{h}{a}(1+m^2)z' + \frac{h^2m^2}{4a^2} = 0$

ie, when $z' = -\frac{hm^2a}{4}$



Inner Panel referred to Reference Plane

Let the corners of the inner panel (p 24) define a reference plane and let the $x'z'$ plane be a parallel plane through the origin. The slope m is given by

$$\frac{b}{a} = m = \frac{\text{height difference}}{7.5 \text{ ft approx}^*} = \frac{3.125 - 0.781}{7.5} = \frac{2.344}{7.5}$$

*strictly $7.5 \cos 7\tfrac{1}{2}^\circ = 7.436$ $\quad \cos 7\tfrac{1}{2}^\circ = .99144$

$$= 0.3152$$

$\theta = \cancel{17\tfrac{1}{2}}^\circ$	$h = 72$	$\frac{hm^2}{a} = 7.4$
$a = \cos\theta = 0.954$	$\frac{hm}{2a} = 11.8$	$\frac{h^2m^2}{4a^2} = 138$
$b = \sin\theta = 0.301$	$\frac{h}{a} = 76$	$\frac{hm^2a}{4} = 1.66$

Eqn of paraboloid (p. 51)

$$\left[x' - (0.312\, z' + 11.8)\right]^2 + \frac{y'^2}{(.95)^2} = 76 z' + 7.4 z' + 138 \qquad \text{.....(3)}$$

Points on plane	x	y	z	ax / bz	x'	az / $-bx$	z'	$y'=y$
A	7.5	0.98	.781	7.12 / .23	7.35	.74 / -2.25	-1.51	
B	15.0	1.96	3.125	14.2 / .9	15.1	2.97 / -4.5	-1.53	
C	7.5	0	.781	7.12 / .23	7.35	.74 / -2.25	-1.51	
D	15.0	0	3.125	14.2 / .9	15.1	2.97 / -4.5	-1.53	
E	11.25	0	1.953	10.7 / .6	11.3	1.86 / -3.37	-1.51	
F	11.25	1.47	1.953	10.7 / .6	11.3	1.86 / -3.37	-1.51	

Approximate formulae

From (1.1) p. 51, neglecting term in z^2: $\quad z' = \dfrac{a^2x^2 + y^2 - hbx'}{2abx + ha}$

This is not good enough

~~The paraboloidal approx~~

Accurate solution for z' [OVER]

From (11), p.51:

$$a^2x'^2 - 2abx'z' + b^2z'^2 + y'^2 - hbx' - haz' = 0$$

Express as quadratic in z':

$$b^2z'^2 - (2abx' + ha)z' + a^2x'^2 - hbx' + y'^2 = 0$$

$$z' = \frac{2abx' + ha \pm \sqrt{(2abx' + ha)^2 - 4b^2(a^2x'^2 - hbx' + y'^2)}}{2b^2}$$

Shape of Inner Panels

15 Feb 1965

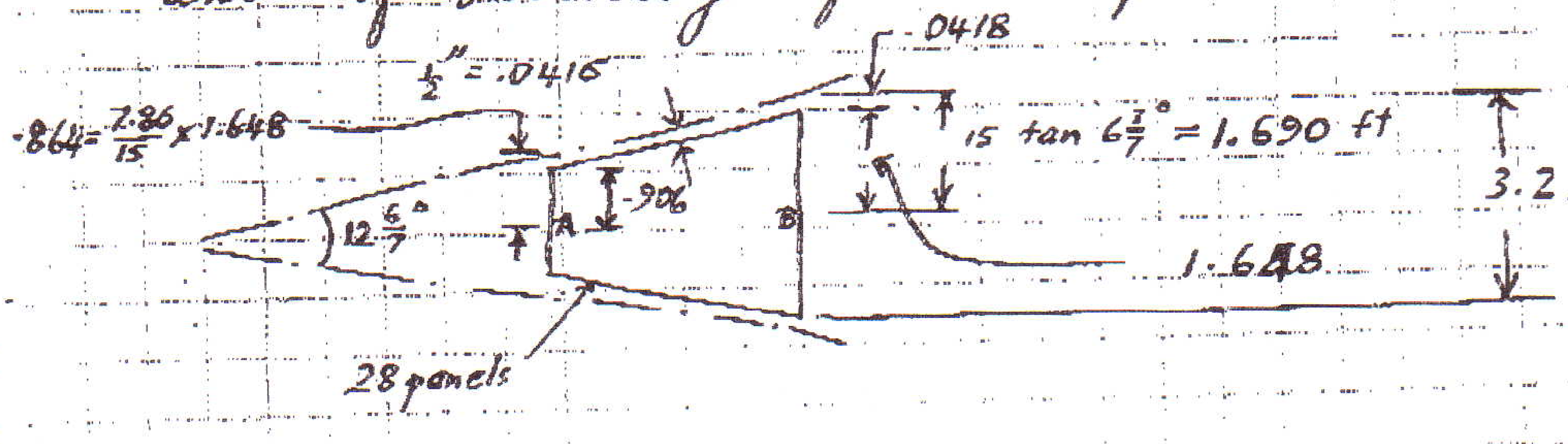
No. of segments : 28

$\frac{360}{28} = 12\frac{6}{7}^\circ = 12^\circ 51.4'$ — sin .22252, cos .97493, tan .22823

$6\frac{3}{7}^\circ = 6^\circ 25.7'$ — sin .11196, cos .99371, tan .11265

$3\frac{3}{14}^\circ = 3^\circ 12.9'$ — sin .05608, cos .99842, tan .05617

Center of outer edge of inner panel is at 15.0 ft radius (B).



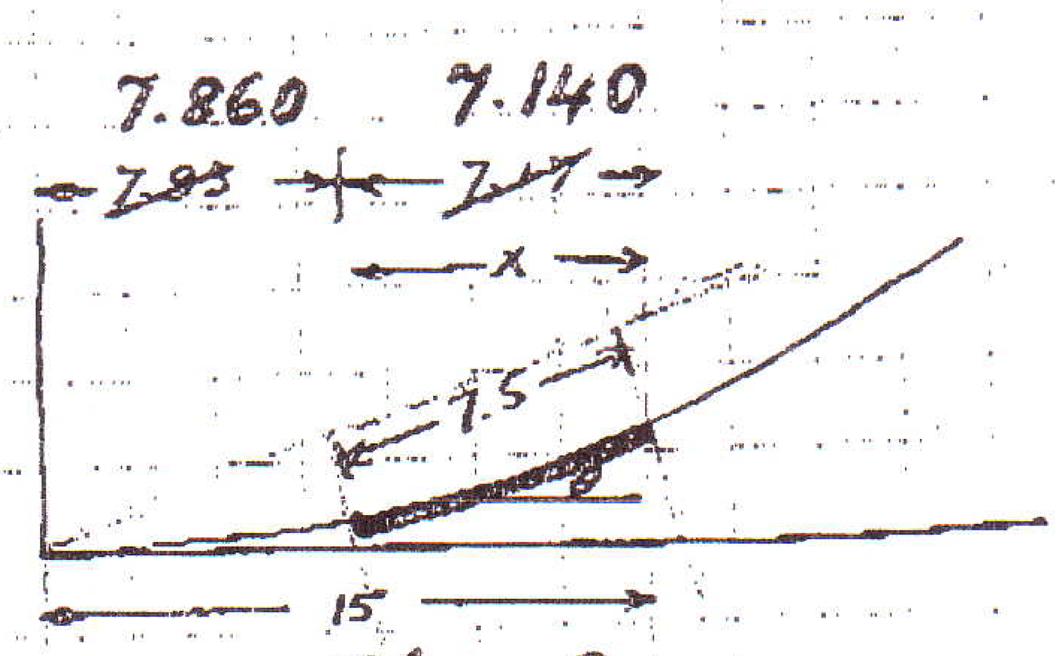
Arc length AB = 7.5 ft

Horizontal projection

$X = 7.5 \cancel{\cos}\ \theta$ $\qquad \tan\theta = \frac{x}{36} = \frac{11.25}{36} = .313$

$= 7.17$ $\qquad \cancel{\sec\theta = 1.048}$

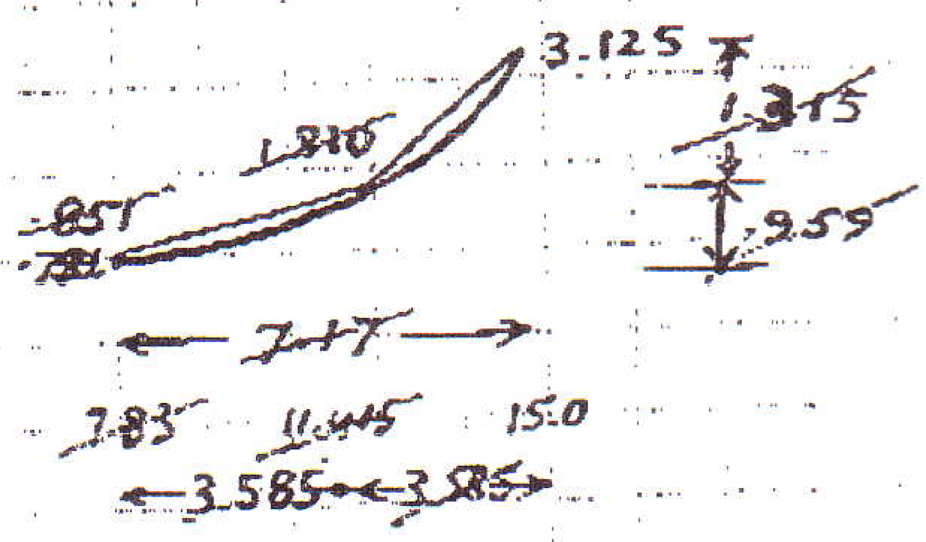
$\cos\theta = .954$



This dimension is based on an approximate arc length. See Machinery's p. 154 for arc length formula – approxn. is $\sqrt{x^2 + \frac{4}{3}y^2}$. I have used the length of the chord. Now verify by adding two short chords

$\sqrt{3.585^2 + .959^2} + \sqrt{3.585^2 + 1.315^2}$

12.852	12.852
.920	1.729
$= \sqrt{13.772}$ +	$\sqrt{14.581}$
= 3.712 +	3.819



= 7.531 Too long by .03 ft. Hence correct the value 7.17 to 7.140

Slope of AB (line joining centers of inner and outer edges of inner panel)

$m = \frac{y_B - y_A}{7.14}$ $\qquad y_B = 3.125 \qquad y_A = \frac{7.86^2}{72} = \frac{61.78}{72} = .858$

$= \frac{2.267}{7.14} = 0.3175$

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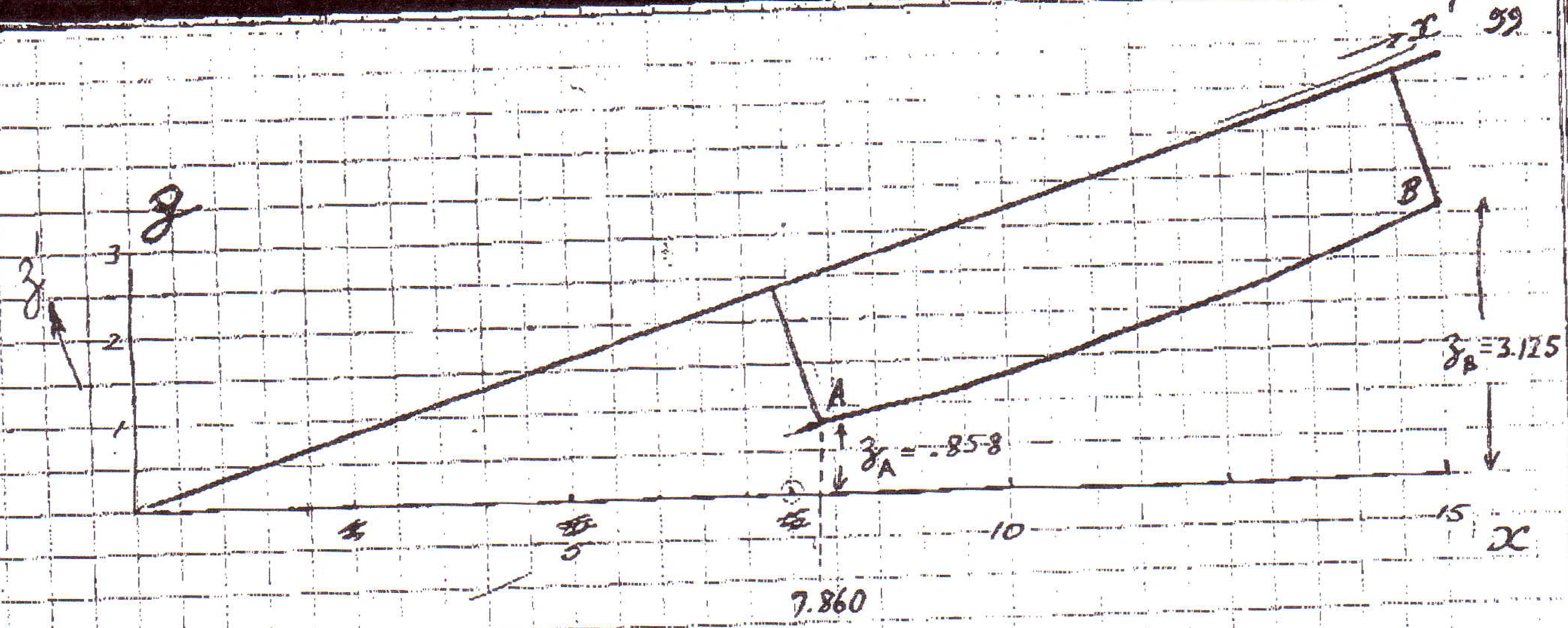

$$x'_B = a \times 15 + 3.125\, b$$

$$x'_A = 7.860\, a + .858\, b$$