Accelerated Program in Elementary School Mathematics The First Year[1]

Patrick Suppes and Duncan Hansen

Institute for Mathematical Studies in the Social Sciences
Stanford University

Accelerated programs in mathematics for gifted children can play an important role in the curriculum. Yet, an extensive review of the literature in the field revealed that few studies of such programs have been undertaken.[2] In response to this lack of information, the Institute for Mathematical Studies in the Social Sciences has initiated a longitudinal study of 40 gifted elementary-school children, who will be accelerated through the mathematics curriculum.

Several factors make schools reluctant to initiate such programs. First, if an accelerated course for gifted primary-grade children is begun, the school will want to avoid returning students to the conventional curriculum for their grade level. Thus, the school is committed to a longitudinal program. A second problem is the shortage of funds for special teaching staff, space, and the development of special curriculum materials. Also, identifying and selecting gifted students at this early level is difficult, especially since students may display differential aptitudes across curriculum areas. Finally, there may be school-parent problems resulting from such a special program. In attempting to deal with some of these difficulties, the project is providing an opportunity for long-range study of an accelerated program. In this article, we describe the first year of the project, which concerned the mastery of mathematical concepts by bright six-year-olds during their first year of instruction.

The results of the 1963-1964 project include parametric data on daily, weekly, and yearly rates of concept acquisition; information about interrelationships between successive mathematical concepts in the curriculum;

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[2]See Carter, 1960, for some of the studies reviewed.

and some insight into the range of topics that can be effectively studied by gifted elementary-school students. Also, our testing included frequent assessment of the personality and social development of the project students.

1 Selection Procedure

The project staff and the school personnel developed the selection criteria. Initially the factors considered were class size, class time in the daily schedule, transportation arrangements, costs, location of the class, teaching personnel, and evaluation procedures. We jointly decided to teach small classes in four contiguous elementary schools that had high proportions of upper-grade students with IQ scores of 130 or over. In addition, we agreed to have the principals and classroom teachers judge the emotional and social maturity of each prospective student in order to eliminate any immature children from the group. To avoid excluding potentially bright mathematical students with differentially low verbal aptitudes, we applied a mathematical ability measure prior to administration of the intelligence measure.

Testing started during the second week of October, 1963. The *New York Test of Arithmetical Meaning* (World Book, 1956) was administered in groups to 133 children who had been nominated by their classroom teachers as possible participants. This test has a reported correlation of $r = +.62$ with the *Otis Quick-Scoring Mental Ability Test* for the normative group. For the 133 children tested, 58 had scores which ranked above the 70th percentile of the 17,000 second-grade pupils who constituted the normative population for the test. The manual interprets this performance as "Better-than-average mastery of arithmetic meanings; adequate conceptual background for more advanced work." These results are presented in Table 1.

We then administered an intelligence scale to these 58 children. Thirty-four students were given a *Stanford-Binet Intelligence Scale, Form L-M* (Houghton Mifflin Co., 1960), by qualified psychometrists. The other 24 were given the *Lorge-Thondike Intelligence Test, Level 1* (Houghton Mifflin Co., 1957), either individually or in small groups. These results are presented in Table 2. The correlation between the arithmetic test scores and the intelligence quotients for the 58 children was $r = +.425$.

A score ranked above the 70th percentile on the *New York Test of Arithmetical Meanings*, plus an intelligence quotient of 120 or above, qualified 43 children for the special classes. In view of the positive correlation between the two measures, and the time required for additional testing, we decided

Table 1: Group Results from New York Test of Arithmetical Meaning

Part I. Pre-Measurement Concepts[3]

Group	Mean	S.D.	Range
A	29.95	1.23	27-32
B	29.22	1.35	27-32
C	27.20	2.24	21-32

Part II. Numerical Concepts[4]

Group	Mean	S.D.	Range
A	31.28	2.18	27-36
B	30.56	1.62	27-33
C	21.89	5.11	6-31

Total Scores

Group	Mean	S.D.	%ile Rank of the Mean Value
A	61.23	2.63	87
B	59.33	1.84	78
C	49.09	6.71	31

Group A: Children who are participating in the program ($N = 40$)
Group B: Children who were given Intelligence Tests but were not accepted as participants ($N = 18$)
Group C: Children who were not given Intelligence Tests ($N = 75$)

Table 2: Group Results from Intelligence Test

	N	Mean	S.D.	Range
Participating Children	40	136.9	12.68	122-166
Children not Selected	18	113.3	7.07	105-126

Table 3: Educational and Occupational Status of Participants' Parents

Education Level	Mother	Father	Parents' Occupational Role	Number
Graduate or Professional School	9	26	Medical Profession	6
			Lawyer	3
			University Professor	6
College	26	13	Accountant	4
			Business Executive	9
High School	5	1	Engineer	8
			Salesman	2
			High School Teacher	2
			Supervisor of Maintenance Crew	1
			Secretary	1
			Housewife	38

that these 43 children adequately represented the four schools considered. Three children were eliminated from this group upon recommendation of their teachers and principals, because of immature classroom behavior. The parents of the remaining 40 children agreed to the participation of their children.

When the classes were initiated in early December of 1963, there were 18 girls and 24 boys in the group. These were distributed in groups of 12, 10, 10, and 8 respectively in the four schools. One child was withdrawn by his parents in early March.

1.1 Sociological Characteristics of the Participants' Families

Available information regarding the children's families was collected from school records. Homes in the neighborhood surrounding these four schools range in value from $20,000 to $60,000. No actual evaluation was made of the participants' homes; but prior studies that utilized students from these schools have usually classified the socio-economic level of the area as upper-middle-class. The educational and occupational status of the children's parents substantiate this upper-middle-class rating. These data are presented in Table 3.

The mean number of children in the participating families is 2.98 children. The distribution of number of children in each family and the ordinal position of the participant are presented in Table 4.

Table 4: Distribution of Family Size by the Ordinal Position in the Family

	Family Size					
	1	2	3	4	5	Total
Oldest	1	5	2	0	0	8
2	-	-	10	2	1	1
3	-	-	-	3	0	3
Youngest	-	6	5	5	0	16
Total	1	11	17	10	1	40

2 Class Curriculum and Procedure

During the academic year 1963-64, the children worked individually in the *Sets and Numbers* elementary-school mathematics program written by the first author, under three qualified and experienced teachers. The range of mathematical concepts in these texts is a rich one compared to most other material currently available. The use of set theory in a simplified manner and the early introduction of geometric concepts are probably the two most salient features.

The concepts introduced in these workbooks may be classified under the following seven categories: (1) sets; (2) operations on sets; (3) numbers; (4) operations on numbers; (5) applications; (6) word problems; and (7) geometry. The specific sequence of concepts is presented in Table 6 below. The table shows the curriculum only through the second grade, since only the three most proficient students were working in Book 3A at the end of the academic year. In addition to the geometry material found in the *Sets and Numbers* texts, the curriculum included *Geometry for Primary Grades, Book I* (Hawley and Suppes, 1960), which develops elementary geometric constructions.

Since the subgroups were approximately ten in number, the teaching staff decided to organize the class routine on an individual basis. After an initial introduction to the first section of the *Sets and Numbers* text, each student came to his teacher for explanations of clarifications as he desired. In many cases the students were able to do all the problems in a section without help, by studying the instructions and examples presented in the text.

Each student's work was corrected immediately after class, and a daily log of problems attempted and errors committed was maintained. The first task for the student on the following day was to correct all errors, with help if desired. Occasionally, a student made consecutive errors on the same problem; then, the teacher reviewed the concept with the student. Each student worked at his own rate, except that the groups were started together again on the geometry text.

The class sessions lasted approximately 30 minutes, four times a week. The students performed all of their work in class, except for the review section in the first part of Books 2A and 3A. Because self-pacing provoked some student cross-comparisons, the teachers made a decided effort to minimize the competition. In the months of May and June, the groups word on simple word problems together in order to establish a sense of group activity.

During the month of July, 1964, 29 of the children participated in small-group works that explored topics in plane and solid geometry, logic, isomorphism of 2 x 2 matrices.

The topics in geometry included lines of symmetry, points, line segments, and concave and convex figures; the concepts inside, on, and outside a closed figure; triangles, quadrilaterals, and pentagons; and faces, edges, and vertices of solid figures, in particular, prisms and pyramids.

The focus of the exploratory work with logic was to familiarize the children with elementary deductive techniques of sentential logic. Three rules of deduction, the "if...then" rule (ponendo ponens), the "or" rule (tollendo ponens), and the "if...then-not" rule (tollendo tollens), were used in deducing conclusions from premises in simple proofs. Many of the exercises required the use of all three rules, either individually or in combinations. At all times the intuitive understanding of the rules was emphasized. Eventually the children learned the techniques of symbolizing the sentences required in the proofs. The following example indicates the nature of the proofs and symbolization.

Sentence form	Symbolic Form
(1) Brian is at Oliver's house or he is with Brent.	(1) 0 or B
(1) If he is with Brent, then he is playing checkers.	(1) If B, then C
(1) But Brian is not playing checkers.	(1) Not C
(1) Is Brian with Brent? No.	(1) Not B 2, 3, IFN
(1) Where is Brian? With Oliver.	(1) 0 1, 4, OR

Work on the mathematical concept of isomorphism explored the extent to which gifted children can recognize sameness of structure of two systems. The class discussed machines which accept balls of certain colors as inputs and release output balls of one of the input colors. For example, if the input balls are either black or red, the following matrix represents a typical machine.

	B	R
B	B	B
R	R	R

Initially we concentrated on familiarizing the children with the matrix representation of the machine. A small machine was built to provide an actual concrete instance; it could produce all 16 possibilities by appropriate setting of switches. Then we considered when two finite sets are isomorphic under a single binary operation. This question was introduced by asking the students when two machines, represented in matrix form, were isomorphic. Also, we considered matrices for balls of three colors; in this case the number of possible matrices is quite large, being 3^9.

3 Project Results, 1963-64

The rate at which the project students worked problems and proceeded through the curriculum was remarkably high, exceeding our initial expectations. There was a consistently high rate of daily performance, with the group daily mean of problems completed being 154.6. For the 26 weeks of instruction, the mean performance of the group was the completion of approximately one-and-three-quarter years of the curriculum. Figure 1 depicts the overall performance in terms of a group cumulative curve. A calibration of the number of problems in terms of the *Sets and Numbers* text material is shown on the ordinate. The mean number of problems in error for the 26 weeks was 2.8 percent. Table 5 presents the biweekly acquisition and error performance of the group.

The group rates according to sections in the text material are presented in Table 6. The results for sections starting in 2A through 2B reflect decreasing numbers of students, since only the more proficient students worked problems in Books 2A, 2B, and 3A. The phenomenally high daily rates are evident in this data too. The only sections where the rates fell below 100 problems per day (Book 1A, Sections I and II, and *Geometry for Primary Grades* reflect restricted work time, when the teacher spent half the class period for general orientation and group discussion.

Figure 1: Curriculum acquisition curves for the total group, fastest student, and slowest student during the twenty-six weeks of the first year. Plot shows mean cumulative number of problems completed versus number of weeks of participation.

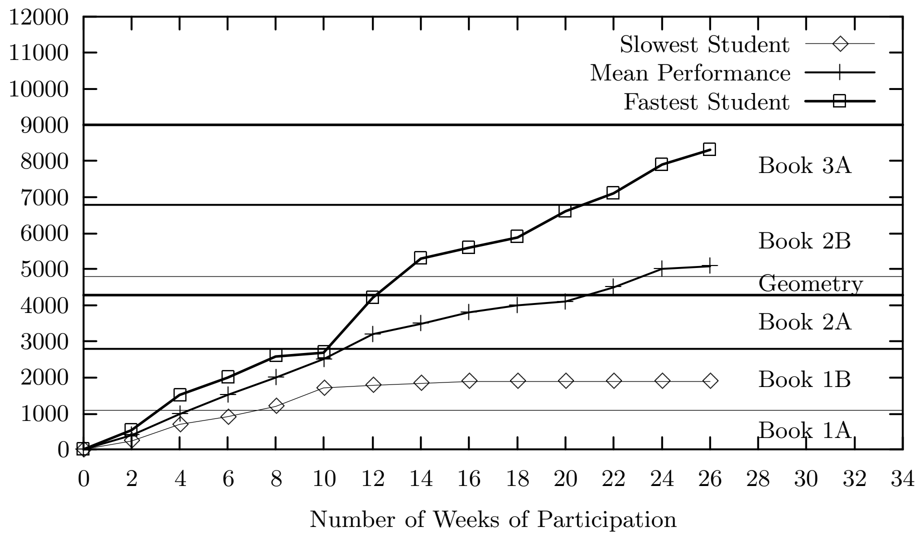

Table 5: Group Acquisition and Error Rates for Two Week Periods

Weeks	Mean Number of Problems Completed	S.D.	Mean Number of Problems in Error	S.D.
1-2	491.3	252.8	11.9	11.8
3-4	525.9	247.5	10.9	8.6
5-6	382.2	175.8	10.1	8.7
7-8	488.4	282.8	11.7	7.3
9-10	500.1	320.8	11.3	6.3
11-12	680.7	445.6	15.6	9.0
13-14	467.0	306.1	15.2	11.5
15-16	265.1	286.7	11.2	8.1
17-18	270.4	239.7	10.1	6.3
19-20	190.3	130.1	5.3	5.2
21-22	297.3	188.5	7.5	7.0
23-24	280.0	254.9	6.7	7.1

Table 6: Daily Acquisition and Error Rates for Various Sections in the *Sets and Numbers* Material

Book Section	Mean Number of Problems Performed Daily	S.D.	Mean Number of Problems with Errors	S.D.
Book 1A				
1. Concept of Set	59.5	30.4	.9	.6
2. Union of Set	87.3	39.7	1.8	.9
3. Concept of Number	126.2	59.3	2.1	1.7
4. Concept of Addition	119.7	50.4	3.0	2.9
Book 1B				
1. Place-value System	110.9	48.1	2.6	2.1
2. Concept of Difference of Sets	118.2	67.2	2.9	2.2
3. Concept of Subtraction	132.2	73.3	3.4	2.1
4. Applications of Numbers	120.0	63.0	3.6	2.6
5. Simple Geometric Concepts	146.5	86.3	3.3	2.4
Book 2A				
1. Review	242.8	125.9	4.0	2.4
2. Application to Measure	228.9	153.9	4.5	3.3
3. Place-values to Hundredths	241.4	147.8	4.3	3.1

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Book Section	Mean Number of Problems Performed Daily	S.D.	Mean Number of Problems with Errors	S.D.
4. Concept of Subset	205.1	154.7	3.7	3.6
5. Concept of Less than	207.5	148.4	4.5	4.0
6. A Set Defined by a Property	212.4	138.5	5.0	4.1
Book 2B				
1. Concept of Multiplication	192.0	83.11	4.7	2.0
2. Concept of Set Abstraction	215.5	125.3	4.9	3.6
3. Fractions	172.0	141.7	6.3	3.4
4. Word Problems and Equations	160.3	111.1	4.6	2.8
5. Carrying in Addition	223.0	111.9	5.8	1.9
6. Intuitive Geometry	200.0	103.3	5.4	1.6
7. Intersection of Sets	148.3	73.4	3.6	1.83
Geometry for Primary Grades				
1. Lines	27.9	21.3	1.7	1.1
2. Circles	28.5	17.4	2.4	1.8

For the initial period of instruction in Book 1A, we have anecdotal reports indicating that the children did not believe the instruction to proceed at their own rates. The adjustment to self-pacing occurred within the first two weeks. The lowered rates for the geometry material are somewhat deceiving, in that each geometry problem requires about five times as many responses as the *Sets and Numbers* type problem. Thus these results are really comparable to those of other sections.

Even more surprising is the extent of variation in rates among individual students. In Figure 1, the individual cumulative curves for the most and the least proficient students are plotted, in addition to the group curve. At the end of the academic year, these two bright children were separated by almost a year and a half of the curriculum. Figure 2 presents the mean percentage of problems completed that were in error. Again, the most proficient student had a consistently lower error percentage in comparison to the least proficient student. The standard deviations presented in Tables 5 and 6 also reflect the large magnitude of individual difference; they represent approximately 50 percent of the value of the mean.

On standardized achievement tests, however, the project students showed extremely low degrees of inter-individual variation. The appropriate *Sets*

Figure 2: Average percentage problems completed which were in error.

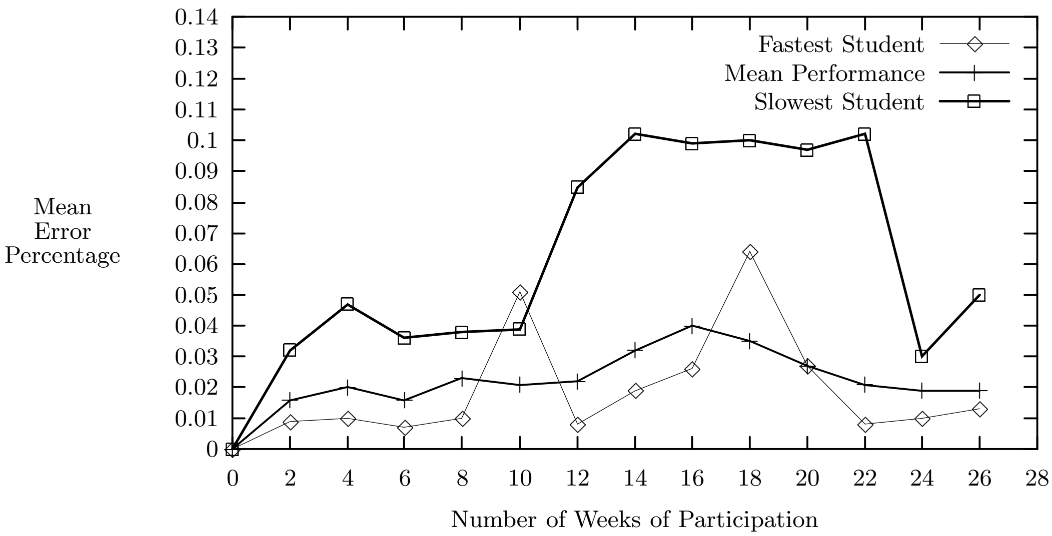

Table 7: Achievement Tests

Tests	Group Mean	S.D.	Total Possible Score	%ile Rank
Sets and Numbers				
Book 1A–Part 1	14.39	0.87	15	99%
Book 1A–Part II	21.23	1.29	22	99%
Book 1B–Part I	20.52	1.11	21	99%
Book 1B–Part II	17.72	2.05	20	99%
Book 2A–Part I	38.11	2.98	40	99%
Book 2A–Part II	32.89	1.31	34	99%
Book 2B–Part I	27.11	2.09	30	98%
SRA Achievement Series				
Reasoning	40.67	4.78	44	99(80)[5]
Concepts	34.80	4.15	37	99(80)
Computation	49.21	4.64	51	99(70)
Greater Cleveland Achievement Tests				
Part I	17.79	0.52	18	N/A[6]
Part II	25.82	1.57	27	
Part III	24.10	3.06	27	
Part IV	23.87	2.84	28	

and Numbers achievement test was administered two weeks after a child finished the particular text material. The SRA tests, Achievement Series, levels 1-2, and the Greater Cleveland Achievement Tests, Grade 1, parts I to IV, were given in the last week of May. The results of these examinations are presented in Table 7.

The achievement test results also indicate the high level of performance of the students. For example, the SRA tests included test items that wee novel in format and employed unfamiliar notation. Even though the children were given no special instruction of training for the test, they still performed in the top quartile in comparison to normative group results that were reported in the test manual.

Although more detailed analyses of the relationships between aptitude and rate variables remain to be done, a general pattern can be cited. The average correlation coefficient of IQ, with the biweekly mean problem rate is $r = +.19$ (range: -.16 to +.51) and IQ with the mean problem rates for curriculum sections is $r = -.19$ (range: -.45 to +.19). The average correlation coefficient of the *New York Test of Arithmetical Meanings* with the biweekly mean problem rate is $r = +.33$ (range: +.01 to +.55) and with the mean problem rates for curriculum sections is $r = +.25$ (range: -.17 to +.58). Thus the selection variables are not impressive predictors of acquisition rates (the restriction of range in the selector variables undoubtedly depressed the covariation). On the other hand, the average correlation coefficient between the mean problem rate for one curriculum section with the succeeding one is $r = +.80$ (range: +.31 to +.96). Thus knowing the self-paced rate in one curriculum section accounts for over the majority of the variation in the next section.

As part of the personality and social development assessment, sociometric peer ratings were obtained both in January and May from the eight first-grade classes which the 39 project students regularly attended. As a group, the project students received significantly higher choice ratings than their non-project peers. More specifically, all the 39 children ranked in the upper quartile on the choice scale. Although there were individual variations in ranks between the January to May comparison, the group remained at the top of the choice scale on both occasions. In addition, the regular classroom teachers rated the project students higher than their classroom peers. On the other hand, we are encouraged to note that no negative effects on the children's social development have appeared.

The project children's performance on the *Torrence Creativity Tasks* (1962) indicate both a wide range of individual differences and a high proportion of superior creative ratings. The children's performance on the Anastasiow Toy Preference Test (1963), a measure of sex identification, indicate a highly bi-modal distribution that in congruent with desired personality development; that is, the selection of one of the two sex identification constellations is highly related to superior academic achievement. The children's performance in the Alexander Interaction Pictures (1952). a measure of anxiety, resulted again in a wide range of individual differences but no incidents of debilitating anxiety. A measure of play interests also reflected a typical range of leisure time activities consistent with normative personality development for this age group. We again are encouraged by the positive results of the personality assessment but are going to continue this line of inquiry in order to insure the early detection of any negative developments. (A more detailed report on the personality and social development of the project children has been prepared by Professor Pauline S. Sears and her associates, who are in charge of this phase of the project.)

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